SEP 23 1921

Vol. XXI. No. 7 Whole No. 180

OCTOBER, 1921

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SCHOOL SCIENCE AND MATHEMATICS

A Journal for All Science and Mathematics Teachers

Founded by C. E. Linebarger

SMITH & TURTON, Publishers
Publication Office, Mount Morris, Illinois
CHICAGO OFFICE, 2059 East 72nd Place, CHICAGO, ILL.

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Published Monthly October to June, Inclusive, at Mount Morris, Illinois
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Entered as second-class matter March 1, 1913, at the Post Office at Mount Morris, Illinois, under the Act of March 3, 1879

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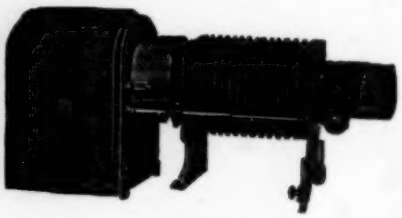

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SCHOOL SCIENCE AND MATHEMATICS

VOL. XXI, NO. 7 OCTOBER, 1921 WHOLE NO. 180

REPORT OF THE WORK OF THE NATIONAL RESEARCH COUNCIL.[1]

BY JOHN M. COULTER,

The University of Chicago.

In the early part of 1916 President Wilson asked the National Academy of Sciences to appoint a National Research Council. The purpose of the Council was to bring into cooperation all of the scientific and industrial activities, with the view of developing our national resources to the maximum. It was a part of the plan of national preparedness. It was natural for the President to turn to the National Academy for the organization of such a Council, since it is the official adviser of the government in scientific matters.

In March, 1916, the Council was appointed and organized for work. In this organization the work was divided among several committees, representing the various sciences. It fell to me to be chairman of the Committee on Botany, and I shall indicate in a general way the kind of activities that this committee developed. It is merely representative of the activities developed by all the committees.

Since the work began in 1916, you will realize that it was before this country went into the war. The work of the National Research Council, therefore, may be considered as representing three periods, namely, the period before the war, the war period, and the reconstruction period.

A brief statement in reference to the pre-war period is as follows. The chief problem faced by the Committee on Botany was the need for an increase of food production. There was a tremendous disparity between the ratio of increase in population and in food production. During the ten preceding years the

[1] Read before the Biology Section of the Central Association of Science and Mathematics Teachers, at Englewood High School, Chicago, November 26, 1920.

population had increased twenty per cent, and essential food production had increased only about one per cent. It will be realized, therefore, that one of the great problems was to enable food production to overtake population.

One of the first things done was to make a census of the investigators of the country, the problems that were under way, and the available equipment. Some interesting situations were discovered. For example, in a certain state one of the best trained physiologists in the country was found at the state university, but with no proper equipment for work. At the state agricultural college there was an abundance of equipment, but no one trained to use it. This census had hardly been completed when the United States went into the war, and an entirely new situation was developed.

In the war period we faced what may be called "emergency" problems. This means that there was no time for prolonged research; in fact, what was demanded was available information rather than research. A few illustrations will be given chiefly to illustrate the nature of the needs that developed. All branches of botany were called upon to answer questions, and the committee really acted as a clearing house of information. The emergency problems may be put under three general heads and were cared for by three different groups.

The first category was food, and numerous questions presented themselves. This food problem may be considered under three heads. The first is food production. Some of the problems already started had a bearing upon this, but had not progressed far enough to be productive. There was a great deal of information available relating to food production, but it was isolated and had never been brought together and focussed upon the problem. For example, one of the important things was the problem of adjustment. People everywhere were trying to grow the same crops, in spite of varying conditions. One of the problems was to adjust the proper crop to the locality and soil in order to secure maximum returns. Another problem was the substitution of well known drought-resistant and disease-resistant races for those that are not resistant. There was a great deal of scattered information upon these subjects, but it had to be brought together and made available. Another problem was that of soil management, chiefly a question of fertilizers. The sources of most of our commercial fertilizers were cut off on account of the war. The potash we had been getting from Germany, and the

nitre from Chile were entirely cut off, and it was necessary if possible to get substitutes for these. In other words, in all these problems the effort was simply to pool available knowledge and apply it.

Another phase of the food problem was food conservation. I do not mean conservation in the ordinary sense in which it was used during the war. It was the kind of conservation that started the disease-resistant surveys in the effort to conserve our crops from destruction. The start in this phase of the work was made by that group of botanists known as pathologists. There came in also another feature, the cooperation with the Bureau of Markets. Thorough studies were made of the marketing of products, and everything—from picking and packing to the final distribution—was considered.

The third and very interesting phase of the food problem came under the head of vitamines, a subject that is coming to the front. It was known that the vitamine content of food was very important, not on account of what is called food value, but on account of enabling one to use food with the greatest efficiency. It is vitamines that may be said to give a man "pep," and the question was raised in connection with the army ration. The problem was to discover a food with the greatest vitamine content that was capable of transportation and could stand storage. There are certain perishable products rich in vitamines, as liver, lettuce, milk, etc., but of course such products could not be transported in an army ration. A specialist in vitamines undertook the work, and found the greatest vitamine content in peanuts; but the problem was that when peanuts are roasted the vitamines are reduced. Before the peanut was advised as a part of the army ration, however, the investigator found a wonderful vitamine content in the soy bean, and this became a conspicuous part of the army ration.

The second general category of problems was timber. The first question asked was in reference to the most desirable timber for the manufacture of airplanes. The ordinary timber used for that purpose is spruce, but there was some question as to whether it would not be necessary to provide ourselves with some other kind that might be used when spruce was no longer available. It was a question of fiber, lightness, and elasticity. Some suitable timbers were found, and in connection with working upon them, the pathologists discovered an interesting situation. A disease might have started in the timber in such an inconspicuous way

that an ordinary inspector would not detect it. If the timber were put in use the disease would spread, and sooner or later the airplane constructed from it would collapse. Many accidents in the early part of the war seemed to have been due to the fact that these little specks of diseased tissue had not been recognized when the timber was prepared.

The committee was asked also to recommend timbers for various other structures in connection with the war, particularly in connection with the various camps in this country.

The third general category was raw products. Few people realize the number of raw products obtained from plants for various uses, such as gums, oils, resins, fibers, etc. The bulk of these raw products had been imported from various parts of the world, chiefly Europe, and of course this source of supply was cut off. It was necessary to discover new geographical sources or new plant sources. A few illustrations will indicate where the pressure was felt. A substitute for red dyes was needed, and in a comparatively short time two or three common plants were found that yielded a red dye that did fairly well as a substitute. The rubber supply offered another problem. The work had been started even before the war in connection with a Mexican rubber plant, for the Brazilian forests were no longer yielding enough. A substitute equal to the Brazilian rubber was not found, but certain substitutes were made available for supplementing the rubber supply. A number of important drugs were called for which are obtained from plants, the supply of which had been cut off, and a number of these were secured. In short, it was found that many valuable sources of supply of such materials exist in our own country, and that we are not as dependent upon foreign sources as was once supposed.

After the war there came the reconstruction period. The results obtained by the National Research Council had been so noteworthy that it became after the war a permanent organization, made up of representatives of the different scientific organizations. It is now continuing the work that had been begun in all branches of science.

In conclusion, I wish to call attention to some of the permanent results to science. The first is that scientific men have learned to cooperate. Our problems have become so complex that only through cooperation can they be solved. The second result is that scientific men and practical men have discovered one another. They have learned that science extends practice,

and practice is suggestive to science. Probably the most important result, however, is the recognition of science as a great national asset, and this means much better opportunities for scientific research. A fourth result is the injection into our teaching programs of courses that will not merely give the content of science, but that will also show that one great mission of science is to serve the public need; in other words, courses that will make the contacts between science and practice, and will not leave science segregated from its human connections.

EFFICIENCY TESTS OF THE NEW CENCO-HYVAC OIL VACUUM PUMPS.

By Professor Chas. T. Knipp and C. S. Palmer,

University of Illinois, Urbana.

The Department of Physics at the University of Illinois has been interested in high vacuum work for a number of years. The new types of mercury vapor condensation pumps that are now generally used in the production of exceedingly high vacua require supporting pumps that will draw a fairly high vacuum of say .01 to .001 mm. of mercury, and maintain it after weeks and months of continued use. Such pumps, if moderately priced, are much in demand at the present time.

Last December, at our request, the Central Scientific Company of Chicago sent one of their new design Cenco-Hyvac oil pumps to our laboratory. We wished to test it for speed and endurance. It was inconvenient at the time to make the speed tests since a pump was sorely needed as a fore pump in the junior electrical measurements work, and later also in research. The pump was put to work at once and has been in almost continuous use three days per week ever since.

Just recently (in April) it was arranged to try it out quantitatively. To this end the pump was connected directly through a short large diameter tube to a 12-liter flask which had attached to it a 500 c.c. McLeod gauge for measuring the pressure, making with the connecting tubes 13 liters as the total volume to be exhausted. The connections and dimensions of the various parts are shown in Fig. 1. In a test like this it is necessary that all connecting joints be absolutely tight. The only two in this setup, the one connecting to the pump, and the other to the McLeod gauge, were protected by heavy oil seals. The test consisted in noting the time and reading the corresponding pressure. Readings were taken at five or ten minute intervals over a

TABLE I.

Cenco-Hyvac Oil Pump No. A112.

13 liter volume RPM = 225		1.3 liter volume RPM = 225	
Time in minutes	Pressure in mm. mercury	Time in minutes	Pressure in mm. mercury
30	.00786	4	.01152
35	.00340	6	.00234
40	.00200	8	.00133
45	.00158	10	.00105
50	.00139	12	.00103
55	.00131	17	.00092
60	.00119	27	.00090
65	.00117		
70	.00115		
80	.00109		
90	.00100		
100	.00098		
110	.00094		
120	.00092		
130	.000911		
140	.00090		
170	.00084		

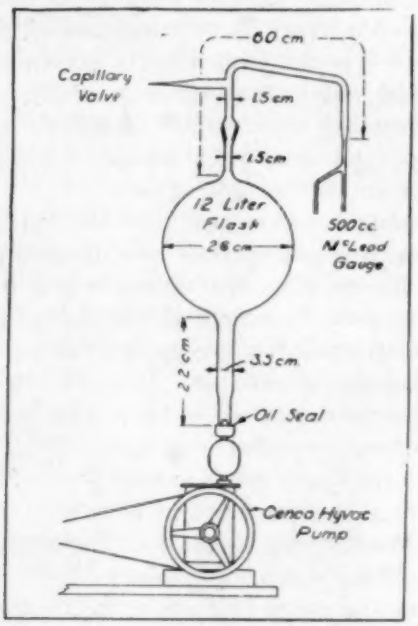


FIG. 1.

period of several hours. The revolutions per minute of the pump were also noted from time to time.

After completing the run with the 13 liter volume it was replaced by a glass tube of about 3.5 cm. diameter, making the total volume now to be exhausted but 1.3 liters—exactly one tenth the former volume. Readings at 5 minute intervals were

again taken. The data relative to this pump with these two volumes are contained in Table I.

Just recently (April 11) the department purchased a second Cenco-Hyvac oil pump. This was at once connected up as shown in Fig. 1, first with the 13 liter volume, then with a 6 liter volume, and finally with the 1.3 liter volume. The running conditions, etc., were exactly the same as in the first pump. The data relative to these three volumes are found in Table II.

TABLE II.

Cenco-Hyvac Oil Pump No. A186.

13 liter volume RPM = 224		6 liter volume RPM = 214		1.3 liter volume RPM = 230	
Time in minutes	Pressure in mm. mercury	Time in minutes	Pressure in mm. mercury	Time in minutes	Pressure in mm. mercury
25	.01260	15	.00564	4	.00832
30	.00480	17.5	.00210	6	.00279
35	.00154	20	.00132	8	.00165
40	.00084	22.5	.00093	10	.00124
45	.00065	25	.00076	12	.00095
50	.00060	27.5	.00073	17	.00079
55	.00057	30	.00070	22	.00062
65	.00056	35	.00067	27	.00051
85	.00056	40	.00065		
105	.00055	45	.00062		
505	.00037	55	.00058		
		65	.00053		
		75	.00051		
		85	.00050		

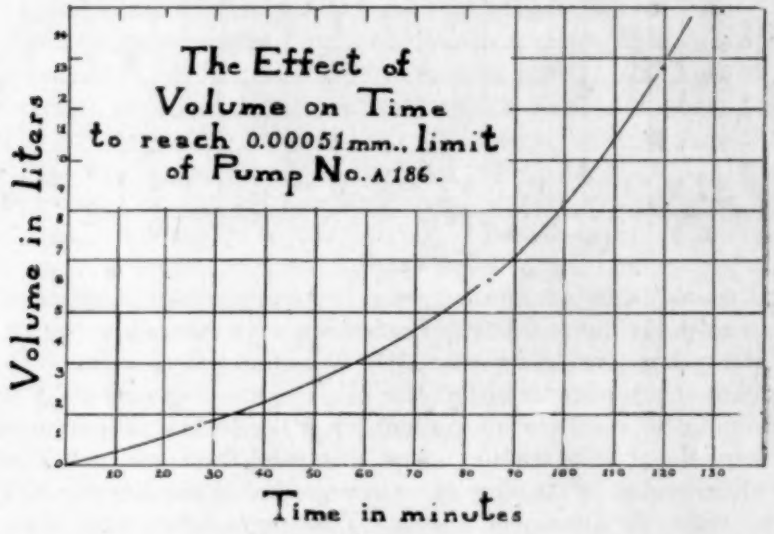


FIG. 2.

The data in Tables I and II are plotted in Fig. 2, in which the crosses represent pump No. A112, received in December, 1920; and the dots pump No. A186, purchased recently. A careful

study of the tables and curves makes further comment seem scarcely necessary; suffice it to say that the limit of pump A112 appears to be at about .00085 mm. mercury, while that of pump A186 is about .00050 mm. mercury. Tags attached to the pumps indicated .001 mm. mercury for A112, and .00055 mm. for A186. Thus the data obtained in our laboratory place the respective limits at pressures that are lower than those claimed by the company. Furthermore, it appears that pump A112 has improved with use.

The relative speed of exhaustion of the two pumps is also shown by the curves. In general the speed is a complex function of the volume exhausted, the time, and the pressure at the beginning and at the end, and need not be considered here.

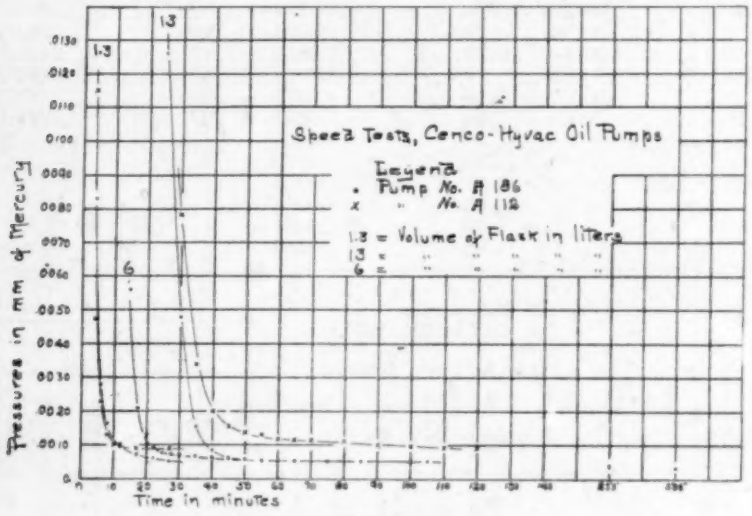

Speed Tests, Cenco-Hyvac Oil Pumps
Legend
Pump No. A 186
x " No. A 112
1.3 = Volume of Flask in liters
13 x " " " " "
6 x " " " " "
Pressures in mm of Mercury
Time in minutes

FIG. 3.

It should be noted, however, that it required about 27 minutes to reach the limit (.00051 mm. of mercury) in exhausting the 1.3 liter volume, while approximately 85 minutes were required to reach the same limit in the case of the 6 liter volume, and 120 minutes to reach the above limit when the 13 liter volume was exhausted. This relation is best shown by the curve in Fig. 3, where volume in liters is plotted against time in minutes required to reach the limit .00051 mm. This curve is for pump A186.

The foregoing data show that this type of oil vacuum pump is quite rapid, and reaches a surprisingly high vacuum. It is therefore well suited to be used as a fore pump to any large throated rapid acting mercury vapor pump, since there is ample overlapping, a necessary condition in any system of exhaustion where fore or supporting pumps are used.

A DREAM COME TRUE.

By Wm. A. Austin,

Fresno High School, Calif.

In School Science and Mathematics for November, 1919, there was an article under the title, "A Flu Dream in Mathematics." This may have appeared to the reader at that time as a mere fanciful dream; but it is no longer a dream in Fresno High School and in several other high schools in California. The plan of teaching geometry as outlined in this dream is now a living reality. It has been in operation here two years and no one thinks of dropping back into the old time-honored plan.

The scheme is the result of an effort to make the study of geometry both valuable and interesting to students and to make the subject matter taught and the method of attack conform to modern ideas of education and life.

Of all the subjects in the high school curriculum geometry is perhaps the only one which has not been revolutionized to some extent, at least, by changing educational policies. The content of the subject and the plan of presenting it in class have not been modified for years. But now, as never before, teachers of this subject, whether they desire to do so or not, must either stand helplessly by and see their beloved subject cast aside as useless, or surrender, to a certain extent, the teaching of Euclidian geometry for formal discipline only and make the work answer the need and demands of modern life.

The greatest step to be taken, the initial step of breaking away from the time-worn rut, will be the most difficult of all.

But once this step is taken, the remaining task will be easy. Our work is an attempt to take this step. For the benefit of the old school, however, we have not neglected logical demonstrations; but we have reversed the method of attack. The method of attack used in our work is founded on fundamental principles which experience has taught us. Our plan of procedure is not new; it is merely the method used by any mind in arriving at a general truth. We claim that the old method, which may be logically sound, does not conform to the usual way of doing things.

The natural method of attacking any problem is inductive. Definite facts and observations are noted. These concrete facts are organized by a mental process and some idea of a general truth is formulated. The mind then searches for a logical demonstration of the apparent truth. When once this proof is clear a general proposition is made and the mind is satisfied.

In accordance with this method we have so planned our work that the pupil must take the following steps in arriving at a general truth. He must, first, make constructions according to specific directions; second, take measurements and perform computations; third, state the conditions of construction and the apparent conclusion; fourth, give the usual formal proof to establish the truth in general; fifth, state the general truth in the form of a proposition. In this manner he develops concepts of the fundamental principles of geometry.

A Class in Plane Geometry at Work.

Some of us have learned by experience that the mind of the high school student can be reached more effectively through the avenue of the muscle than through the avenue of the eye or ear. A student may stare at a construction in the text or on the board, or he may listen attentively to a logical demonstration or to a discussion of a construction by the teacher or fellow student; but unless he makes the construction with his own hands, it does not impress him permanently. Moreover, when he looks at a construction in the book or on the board, he sees it as a completed whole and is bewildered, for he is not able to distinguish the beginning from the end. But when he makes it

with his own hands, following specific direction, step by step, he comprehends the whole as a logical process.

The general plan of our work is as follows:

Equipment. We have two rooms equipped especially for our purpose. Each room is furnished with drawing tables and chairs, instead of rows of desks. Each student is provided with a drawing board, a T-square, a triangle, a scale, a protractor, a compass, dividers, thumb tacks, a folder containing sheets of drawing paper, and a copy of the *Manual.* There are large cabinets in each room. These are used by students to file away in good order their completed work.

Next year when we are in our new building and have forty-five minute periods we intend to have a double period for each class in plane geometry, just as in any laboratory science. We will have two large rooms equipped like the ones we now have. Each room will seat 50 or 60 pupils at once. This will be the purely laboratory room in charge of two teachers, each teacher having charge of half the pupils. Adjoining each large room is a recitation room, into which either teacher may take a group or all of her students for recitation work or for instruction purposes. She may remain in this recitation room until she gets through her work. When she returns to the laboratory room the other teacher may take all of her class or a portion of it into the recitation room and remain as long as is necessary.

Procedure. Students follow directions given in the *Manual* and prepare plates. Each plate consists of one or more constructions and the necessary computations, discussions, definitions, or proofs. Fundamental conceptions, definitions, and theorems are developed on these plates. In making these plates neatness and accuracy are observed. The work is done with a sharp pencil. Some students do the work in ink, but ink is not required. No attempt is made to prepare mechanical or geometric drawings. These plates are made for the specific purpose of learning geometry. Frequently it happens that a student has taken or is taking mechanical drawing. He may possess his own drawing instruments, and if so the accuracy and precision demonstrated by his constructions, measurements, and computations are often very good.

Exercise plates are frequent. They consist of originals, constructions, and corollaries. Students prepare these plates without directions or with few hints. Miscellaneous exercises and problems of practical applications are given in great numbers. Summaries of facts established are given at frequent intervals.

Numerical exercises are introduced at every opportunity to illustrate the therom or corollary demonstrated. These exercises frequently involve algebraic solutions. The teacher uses these exercises and exercise plates to test the student's ability to prepare work independent of directions or suggestions, and to fix in mind the truth of the proposition.

Quantity of Work. Seventy-two plates are required of all students for the year's work. These plates contain definitions, fundamental ideas, constructions, and over eighty theorems and their corollaries. The number of exercise plates prepared, exercises solved and ability shown in the recitations, in addition to the seventy-two plates, determine the student's standing. These regular plates and exercises cover all the work usually given in any text in geometry.

Character of Class Room Work. The period is passed in an informal manner much of the time, the students being engaged in making their plates. The teacher is generally busy the entire period in answering questions, offering suggestions, and giving individual or group instruction as the occasion demands. Students are not only permitted to talk with each other about their work, but they are encouraged to recognize the leader at their table and consult him when in trouble.

Recitations. Formal recitations are held at frequent intervals. These recitations are sometimes by the whole class, but more frequently by small groups, chiefly by tables. It is interesting to note that the student in his demonstration of a proposition talks about things he knows and things he has done. He does not repeat what he has heard some one say, read from a book, or committed to memory.

Home Work. Exercises, exercise plates, and problems are usually prepared outside of class. Reviews for recitations and tests are also done at home.

The work outlined in our *Manual of Plane Geometry* is specific and definite. The student who follows the directions step by step cannot go astray. Without sample drawings made by teacher and without the constructions in any text to guide him, the student is able to complete his drawing, take his measurements, draw his conclusions, and prepare a general proof. The care with which these directions are written enables the student to do this.

Each student may be busy every minute, for he knows just what to do. When he enters the room he may go to his work

at once without waiting for a teacher to set him at work. I entered one class one morning and found all the pupils busy at their work even if their teacher had had trouble with his car and was twenty minutes late. Another time I went into a room to see the teacher. She was not there. She had stepped into the office on business. Not a single pupil knew where she was, and no pupil realized that she was out of the room. This is a common occurrence.

The work is concrete. The student does it with his hands, and hence he is happy. He can see the results of his labor. Geometry is not only a thing to talk about, but it is a thing to do with precision and accuracy. He is proud of the work he does. He treasures these plates, which contain the whole subject matter of geometry, much more than he does any text on the subject. He takes these plates home when completed and shows them to his parents and friends.

The student sees a problem in the process of its construction. In former times he too frequently committed to memory, not only the drawing but the demonstration, to make a good showing before the teacher in class. But when he does it with his own hands, he realizes that the completed whole is a rational process after all. He can, therefore, stand before a class and discuss a subject for he knows what he is talking about. He does not point haphazardly at a drawing and recite words committed to memory.

Each student is permitted to demonstrate his own strength; each student sets his own pace. His work need not be affected by the slowness of some pupils nor by the brilliancy of others. His progress is a function of his own ability. The length of time necessary to complete the required amount of work satisfactorily is in his own hands. He is not permitted to advance till the task in hand is accomplished. No student, therefore, should fail.

Last year we had about 250 students taking this work. This year we have had over 350 in geometry classes. That students may demonstrate their individual ability is shown by the fact that last year we had a class of 20 who completed both plane and solid geometry in one year. This year we will have a class of 30 to do this. The strong student is given all the rope he wants, just as long as he demonstrates that he is getting the work. If a pupil is slow in grasping the subject he is not embarrassed by the speed of others. The slow pupil does not need to keep the

pace set by a teacher or by a class, and thus become bewildered, lost and failed.

The so-called "socialized recitation" may be realized and the hum of industry—not noise—obtained. The teacher as we knew her in the past is useless. The pupil asks the questions now. The teacher must be on the job every minute, suggesting, answering questions, showing how it is done, giving individual or group instruction, and inspecting and passing on the work. The students group themselves at the tables according to their own likes and dislikes, each table forming a sort of unit.

In the plan of this work there is an opportunity to teach, in addition to the subject matter of geometry, correct habits of industry, promptness, and attention to duty. A student who signs up to take this work must be taught to read directions and specifications and to carry them out to the letter. He must be made to realize that school life is work, real work, at real things; he is doing things, definite things; that is just what the world will demand of him later.

A few illustrations of the type of work required of students may be given.

Fig. 1 Plate VII illustrates the idea. The directions given in the *Manual* follow:

The Area of a Triangle. Construct triangle ABC so that angle A equals 110 degrees and the including sides AB and AC are 81 rods and 97 rods. Use the scale, 1 m.m. equals 1 rod.

Use plan of Fig. 2 Plate V and construct the three altitudes of this triangle. Sides BA and CA must be produced. The altitude AM falls within the triangle but the other two altitudes BN and CR lie outside the triangle. If the altitudes are produced they should meet in a point.

The area of a triangle equals one half the product of the base and altitude. Any triangle has three altitudes and a base corresponding to each. Pick out the altitude corresponding to each base.

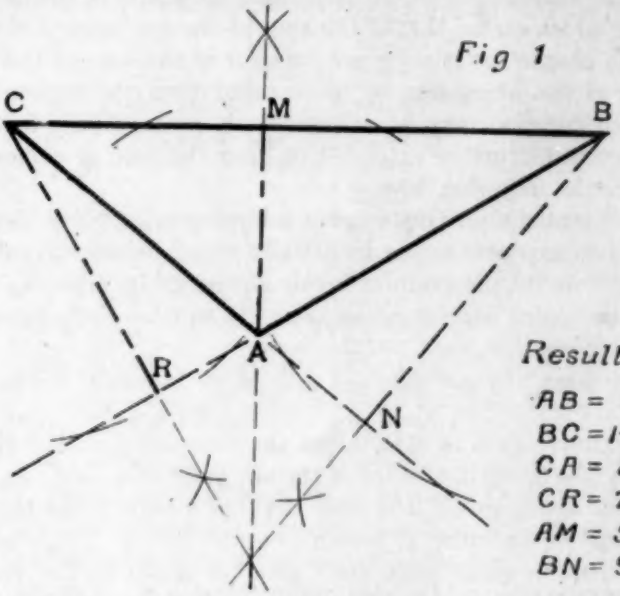
Measure the three bases and the three altitudes. Find the area of this triangle by using each base and its corresponding altitude. This gives three answers, but if the measurements are correct and the drawing is accurately made, these three answers will be equal.

Write the measurements and the computations of the area under the drawing.

The accompanying drawing of Fig. 1 Plate VII is typical of this construction found on students' plates.

On Plate XLIX students are directed to draw a circle. Two or three secants and a tangent are drawn from an external point. The length of each secant and its external segment and the length of the tangent are carefully measured. The product of the whole secant and its external segment is found in the case of each secant. These products are compared with the square of the length of the tangent. This may be tried in two or more circles.

It dawns upon the student sooner or later that if absolute measurements were taken on an accurate drawing, that the product of the whole secant and its external segment is constant, and that the tangent is the mean proportional between the whole secant and its external segment.



Area = ½ AB × RC = ½ 97 × 76.5 = 3710.25
Area = ½ AC × BN = ½ 81 × 91.5 = 3705.75
Area = ½ BC × AM = ½ 148.5 × 50 = 3712.5

Then follows the ordinary proof to establish these facts in general. When this is done the student states the proposition in the form of a theorem. Following this are many numerical exercises whose solution involves the truth just established. A few exercises require the use of algebra.

On Fig. 1 Plate LI students are directed to construct a triangle and to bisect any interior angle, letting the bisector intersect to opposite side at D. They then measure the two sides of the triangle including the bisected angle, and also the two segments of the third side made by the bisector. They find the ratio of the two sides and also of the two segments correct to three decimal places. Following this is the usual geometric proof of the fact discovered by these measurements and calculations.

Note. The author will be pleased to answer any question which the reader may have in mind.

AN EXTENSION OF A PROCESS IN FACTORIZATION.

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In the application of the Remainder Theorem to factorization of algebraic expressions we are assisted in the choice of the factor by a rule which states that if the sum of the coefficients of the expression chosen for trial be not a factor of the sum of the coefficients of the expression to be factored then the expression under consideration may be rejected as it can not be a factor. This rule can be further extended to limit the field of choice of factors in the following way.

The rule stated above rests on the following principle of identities. If two expressions are identically equal, whatever values you substitute for the symbols involved the results given by the expressions under consideration must be equal. For example, let us suppose that,

$$x^n+p_1x^{n-1}+p_2x^{n-2}+p_3x^{n-3}+ \quad . \quad . \quad . \quad . \equiv (x-a)(x^{n-1}+q_1x^{n-2}+q_2x^{n-3}+ \quad . \quad . \quad .)$$

is an identity. $x-a$ is a factor of the expression on the right hand side and hence is a factor of the left hand side also. As the expressions are identical it is clear that the results given by the two factors $x-a$ and $x^{n-1}+q_1x^{n-2}+q_2x^{n-3}+ \quad . \quad . \quad .$ after the substitution must have their product equal to the result given by the left hand side. Thus the result given by $x-a$ after the substitution must be a factor of the result given by the expression on the left hand side.

If you substitute unity for x the result given by the expression on the left hand side and the expression $x-a$ are each the sum of their coefficients.

$$[x^n+p_1x^{n-1}+p_2x^{n-2}+p_3x^{n-3}+ \quad . \quad . \quad .]_{x=1} = 1+p_1+p_2+p_3+ \quad . \quad . \quad .$$

$$(x-a)_{x=1} = 1-a.$$

Hence if $1-a$ be not a factor of $1+p_1+p_2+p_3+ \quad . \quad . \quad . \quad .$ we can infer that $x-a$ is not one of the possible factors.

Put x equal to some other quantity. If the value of the expression $x-a$, the factor for trial, is not a factor of the value of the expression $(x^n+p_1x^{n-1}+p_2x^{n-2}+ \quad . \quad . \quad . \quad . \quad .)$ under factorization after substitution then $x-a$ can not be a factor.

Proceeding in this way we can limit the field of choice. An example will make the method plain. Let us suppose that we want the factors of $30x^4-71x^3-37x^2+130x-24$.

Numbers which will divide 30 without remainder are 1, 2, 3,

5, 6, 10, 15, and 30. Numbers which will divide 24 without remainder are 1, 2, 3, 4, 6, 8, 12, and 24.

The possible factors may now be written as follows:

	1	2	3	5	6	10	15	30
1	$x-1$ $x+1$	$2x-1$ $2x+1$	$3x-1$ $3x+1$	$5x-1$ $5x+1$	$6x-1$ $6x+1$	$10x-1$ $10x+1$	$15x-1$ $15x+1$	$30x-1$ $30x+1$
2	$x-2$ $x+2$		$3x-2$ $3x+2$	$5x-2$ $5x+2$			$15x-2$ $15x+2$	
3	$x-3$ $x+3$	$2x-3$ $2x+3$		$5x-3$ $5x+3$		$10x-3$ $10x+3$		
4	$x-4$ $x+4$		$3x-4$ $3x+4$	$5x-4$ $5x+4$			$15x-4$ $15x+4$	
6	$x-6$ $x+6$			$5x-6$ $5x+6$				
8	$x-8$ $x+8$		$3x-8$ $3x+8$	$5x-8$ $5x+8$			$15x-8$ $15x+8$	
12	$x-12$ $x+12$			$5x-12$ $5x+12$				
24	$x-24$ $x+24$			$5x-24$ $5x+24$				

There ought to be $8\times8 = 64$ pairs (i. e. 128 in number) of possible factors, but the expressions such as $2x-2$, $15x-3$, etc., are rejected since they are divisible by 2 and 3 respectively, which is not a factor of the original expression and also because they are included in $x-1$ and $5x-1$, respectively. There are then only 60 possible factors. The sum of the coefficients of the expression to be factored or the result of putting x equal to 1 in it is $30-71-37+130-24 = 28$.

As this is not zero, $x-1$ is not a factor of the expression.

The sum of the coefficients of odd powers of x is 59. The sum of the coefficients of even powers of x is -31. Thus $x+1$ too is not a factor.

Putting x equal 1 in the above table of factors and underlining those factors which give a result which is not a factor of 28 and rejecting them the following possible factors remain:

$2x-1$	$x-2$	$x-3$	$3x-4$	$x+6$	$x-8$	$5x-12$	
$3x-1$	$3x-2$	$x+3$					
$3x+1$	$5x+2$	$2x-3$	$3x+4$	$5x-6$	$15x-8$		
$5x-1$		$5x-3$	$5x-4$				
$6x+1$		$10x-3$					
$15x-1$							

So that the field of choice now extends only over 22 factors. It is therefore reduced by $60-22 = 38$.

Put $x-3 = 0$, or $x = 3$ in the expression. The value of the expression is 546. As this is not zero, $x-3$ is not a factor.

The values of the possible factors are given in the following table:

5	1	0	5	9	−5	3	
8	7	6	13	9	37		
10	17	3	11				
14		12					
19		27					
44							

Cancelling those factors which give a result which will not divide 546 only seven factors remain.

$5x-1$	$x-2$ $3x-2$	$x+3$ $2x-3$	$3x+4$	nil	nil	$5x-12$

Put $x+3 = 0$, or $x = -3$ the expression equals 3,600. As this is not zero, $x+3$ is not a factor. The values of the possible factors are:

−16	−5 −11	0 −9	−5	nil	nil	−27

Cancelling those factors which give a result which will not divide 3,600. Only four factors remain.

$5x-1$	$x-2$	$2x-3$	$3x+4$	nil	nil	nil

Now let us try $x-2 = 0$, or $x = 2$. The expression equals 0. Therefore $x-2$ is a factor.

Then try $5x-1 = 0$ or $x = 1/5$. The expression equals 0. Therefore $5x-1$ is a factor. Now try $2x-3 = 0$, or $x = 3/2$. The expression equals 0. Therefore $2x-3$ is a factor. In the same way $3x+4$ can be proved to be a factor.

As the expression is of the fourth degree it is clear that it will not have more than four first degree factors so that if the expression has a fifth factor it must be a numerical one. Let K be the factor. Then $30x^4-71x^3-37x^2+130x-24 = K(x-2)(5x-1)(2x-3)(3x+4)$.

By comparing coefficients we have

$$30Kx^4 = 30x^4, \text{ or } K = 1.$$

This method is not, of course, successful when the expression has no first degree factors. However, in any case you can by applying this method at least definitely tell whether or not the expression will have any first degree factors so that you can then apply the inspection method.

THE INTELLECTUAL RISE IN ELECTRICAL SCIENCE.

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The remarkable growth of the sciences in recent years has been accompanied by a commensurate unfettering of the human mind. The man who tries to stand aside and look back at the intellectual progress of the past is bewildered by the diversity of activities of the mind and by the complexity of its achievements. Particularly is this true of the person untrained in science who tries to fathom the abstruse depths of mathematical physics or of modern electrical theory, and yet it is the achievements in just such fields as these which best measure the intellectual rise of man.

The unparalleled development of electrical science has resulted from accurate, painstaking, and impersonal observation of the phenomena of nature, interpreted by the most intricate and logical methods of mathematics. To a high degree the subject has been freed from the bonds of dogma and superstition, and hence to a similar degree must it represent the power of the intellect to deal with the raw facts of experience.

The change in thought in the past hundred years has been remarkable and revolutionary. The magnet had been long looked upon as a thing possessed of the devil; static electricity was considered an evil omen and current electricity was taken by many as being a direct revelation of divine power. A few years ago, a writer upon the subject said that the intellectual rise reached a climax when lightning, electricity and magnetism were identified as manifestations of one and the same source of energy. This was a climax to be sure but it only marked the beginning of a greater age in which the unfettered mind of today is revealing its higher capacities.

The wonderful progress of the past century has been made in spite of three great conflicts of science with preconceived notions. The greatest of these was, of course, the conflict with theology, or rather the church. Next to this came the conflict against scientific dogmatism, and the preconceived ideas of the scientists themselves. Then also has science continually had to fight the popular misunderstanding of it own aims.

A little more than a hundred years ago Lagrange declared that science should be independent of theology. Since that moment science has been gradually achieving her freedom. From the dominion of the church she passed into open conflict

with it during the latter nineteenth century, and is only now emerging from the darkness of one prejudice or the other. The problems of life and nature and God have ever been in the mind of man. In early Greece the same questions arose which survive today, but it has become more and more apparent that the physicist has to deal with the actual facts of experience, and not with vain speculations as to what may lie back of these facts.

The rational process of intellection is one of accurate observation of the real data of experience, followed by a correlation of this with our main body of knowledge, upon which basis inferences may be drawn, tested and applied. This is the so-called scientific method, and it is the same whether applied to the complex problems of life and organic growth, or to the various problems of the inorganic world from the birth of atoms to the birth of worlds. If his result is to be reliable, the scientist must steer clear of speculation and prejudice.

Since the time of Lagrange scientists have been zealous in attempting to divorce science from anything approaching speculation. For this reason they refused to accept many of the mathematical inferences of the time, as being in a manner speculative. The many disagreements between the physicists and mathematicians are very amusing, when one considers the close relation and harmony which should exist between the two groups. However, such disagreements frequently presage some great intellectual advance.

One of the greatest examples of the influence of preconceived ideas is seen in the development of the theory of gravitation. This also had an important bearing on the development of electrical theory. Newton's theory of universal gravitation, like many truly great theories, was slow in gaining ground. Though enunciated in 1666, Whewell claimed that prior to 1728 there was not a score of Newtonians outside of England. When the pendulum of favor did swing in his direction, it over-swung, as is usually the case. In twenty-five years more, gravitation was seized upon and swallowed whole. Bernoulli, Boscevitch and the mathematicians immediately interpreted it to act through space at any distance without an intervening medium. In this they were precursors of Einstein who disregards the ether in his theory of relativity. Einstein, however, does not prove its non-existence, but merely shows that as far as his theory is concerned it may be disregarded. The more elastic viewpoint we have reached is a distinct advance in that it does not shut

the door on the problem, but leaves it open. It has always been intellectually unsatisfactory to try to conceive of action at a distance and the older mathematicians had drifted far from Newton as is clearly shown in a letter from him to Bentley.

"That gravity should be innate, inherent and essential to matter so that one body may act upon another at a distance through a vacuum without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity that I believe that no man who has in philosophical matters a competent faculty of thinking can ever fall into."

No doubt Newton would have been sadly shocked if he could have known that before the end of the eighteenth century action-at-a-distance was a fully accepted doctrine. Exactly the same thing recurred at the time of Faraday when the interaction of electricity and magnetism was first discovered.

From the time of Faraday to the present there has been a continual change in methods of scientific thought and attack. Early science had to deal with things which are visible and with the immediate data of sense perception. Naturally from the historical and experimental viewpoint mechanics became the most fundamental branch of physics though from the viewpoint of metaphysics it may be the most superficial of all.

Early mechanics was at a loss to cope with the problems of fluids. The phenomena were too subtle to yield to direct mechanical analysis, so it was necessary to build a simple mechanical model in conception that would produce the same phenomena. Thus grew the kinetic theory of gases, and the more remote facts of experience were interpreted in terms of the immediate. This was a synthetic process in which the theory was built to fit the phenomena. Obviously it led to a mechanistic conception of natural phenomena. Theories were built up from mechanical principles only.

This synthetic process has given way more and more to analytic processes in science, of which the most striking achievement is the present theory of relativity. Conceptual models will continue to be formulated but the farther the boundaries of science are pushed from the realm of gross sense data, the less can these models be depended upon to represent reality, and more and more must we resort to subtler methods of analysis. This shifting viewpoint is marked by three great milestones, the development of Maxwell's electro-magnetic ether, the discovery

of the electron, and the formulation of the Einstein theory. Experimentally we have the discovery of electro-magnetic waves, the discovery of the electron, and the discovery of the deviation of light waves in a gravitational field.

Franklin discovered lightning and electricity to be identical in 1752, and in 1831 Faraday discovered the facts of electro-magnetic induction. In the intervening years, some progress had been made but the theories were totally unable to explain the new facts of electric and magnetic interaction. It is not surprising that the mathematicians resorted to their preconceived ideas of centers of force and action-at-a-distance.

Faraday was not a mathematician, but he possessed the true mathematical mind, and he freed himself from the preconceived notions of the time. Though unable to formulate the mathematical conditions of the ether which he conceived must transmit the electro-magnetic forces, he clearly formulated a mechanical concept of their simplest conditions. This was diametrically opposed to the current theories. No matter how crude his concepts were they formed the foundation for the later work of Maxwell and others and for the theory of today. As Lord Kelvin has said: "Since Faraday first offended physical mathematicians with his curved lines of force, many workers and many thinkers have helped to build up the nineteenth century school of plenum, one ether for light, heat, electricity and magnetism."

Such were the events that led up to the electrical era, ushered in by Maxwell. The first half of the eighteenth century was given over to the struggle concerning the validity of Newton's law, followed by the question as to the nature of transmitted forces. However, "Before Faraday's death in 1867 he had succeeded in inspiring the rising generation of the scientific world with something approaching faith that electric force is transmitted by an ether, of which for forty years the world had believed light and heat to be transverse vibrations."

The facts of electro-magnetic induction waited forty-two years for a Maxwell to interpret and at last he came. It takes ingenuity to perform experiments, and to perform them with a definite purpose takes much more than that, but it takes genius of the highest order to draw away the curtains of the mind and disclose the future as did J. Clerk Maxwell.

In 1873 Maxwell reversed the old problem and set himself the task of determining what properties a hypothetical medium should have in order to transmit electro-magnetic forces in the

way that had been observed by Faraday and others. These he determined in the form of his historical mathematical equations, which stand today as the foundation of modern electro-magnetic theory, and one of the greatest achievements of all time. Maxwell showed directly that an electric wave in such a medium as he had hypothecated would travel with the speed of light. The later experiments of Hertz proved this and identified light waves as being electro-magnetic in nature.

It had been a comparatively small step to identify electricity, magnetism, and lightning as manifestations of one and the same source of energy. The new conception that light should be identified with these was startling. Light had always been set aside as super-mechanical, and apart from other natural phenomena. The new classification of such an old and seemingly separate branch of physics was revolutionary indeed, but it was only the beginning of the new vision of physics in which all the so-called separate branches have reunited, and have brought us an altogether new conception of the physical universe.

The ether which Faraday had dimly seen afar off became definite, and not only that but it turned out to be nothing more than the optical ether which had been invented so laboriously before, except that it was less mechanical. McCullagh and others had tried to construct conceptually an actual working model of the ether. Even Maxwell felt he must justify his theory the same way and he built what was at first a very definite conceptual model. Later on he placed less stress on the model and more on the relations he had sought to express. As Schuster well says, "In estimating the support which Maxwell's views have received from experiment, we must distinguish between the fundamental assumptions on which Maxwell based his investigations and the mathematical formulae which were the outcome of these investigations. It is clearly the mathematical formulae only which are confirmed, and the same formulae might have been derived from quite different premises." The ether model did not show what actually happens, but only showed a way in which the same result could be produced. Obviously the equations of Maxwell are the real representative truths of the ether. Thus a material mechanical ether becomes more vague and irrational and the ether that remains is a mathematical one. It serves just as well the purposes of scientific explanation, and is necessary in many cases as a working basis for further advance, but mechanically it is a name to cover up

our ignorance, and to express that innate demand of the intellect that objects interact by means of some medium. Maxwell in reversing the old problem made it an analytic one which he only felt it necessary to reinforce synthetically in part. His was the first great analytic achievement in the field of electricity, and it will stand an everlasting monument to his name.

The electron was discovered in 1895. Study revealed it not as an atom of matter but as an atom of energy smaller far than any known atom. That energy exists in localized units, and that matter is composed of these units was further evidence of the unity underlying all things The school of energetics has grown rapidly, and not without cause. Energy is fundamental. Matter and electricity are two of its particular manifestations. Radiation of light which had always been a puzzle appears to be the direct product of electro-magnetic forces within the atom. The electron is the smallest individual particle yet discovered, but the weight of its discovery has displaced mechanics, the historical pretender, from the throne of physics.

The relatedness of all natural phenomena strengthens the belief in the fundamental simplicity of nature, and this relatedness foreshadowed the theory of relativity. At the same time it gave a renewed impetus to model building. Everyone wanted to know how different atoms could be built of similar electrons and how an atom radiates light. Then came the Thomson model atom, and the Rutherford and the Bohr models. The shifting intellectual viewpoint, however, is marked. The Bohr model, which achieved the distinction of predicting a new series of lines in the hydrogen spectrum, broke away from classical mechanics. It was an application of pragmatic philosophy to physical science, in that it was built regardless of classical mechanics, and it is valued for what it has actually accomplished, not for the security of its foundation on the past.

Less and less science builds on the past, and on what it thinks ought to occur. Phenomena must be taken as they come, and analyzed in any manner that will disclose further truths. For a time it may seem that the scientist uses contradictory postulates quite willfully to suit his own convenience. But always at certain intervals simplifying discoveries are made which bring order again out of complexity. The theory of relativity is such. Whatever its ultimate fate may be, it has brought a revolution in ideas which marks the beginning of a new era. Not only are the objects of perception related as we have seen, but the theory

of relativity relates the two fundamental modes of perception, time and space. There may be many times and many spaces, and they condition, and are conditioned, by the objects perceived in them. The difficulties of the theory are great, but its triumphs have been greater. The deviation of light and the irregularity of Mercury's orbit have shown us how it refines our past gross views of gravitation, and the various extensions of the theory show us how it reaches out and deals with facts far beyond immediate sense perception.

The theory of relativity is an analytical one. It takes a principle to pieces and sees what the conditions are upon which it depends. These conditions or relations are the things we can know. Whether or not our models are in any way representative we cannot know. Popular opinion has always confused these two notions and has not understood the aim of science.

Science can never explain what is back of phenomena. It describes merely how phenomena occur, and the more analytic it becomes the better it can describe. The intellectual unfolding attending the development of electrical science has followed steps as definite as they are striking. The mind of man has climbed steadily upward, and though the great Tyndall went a little far in saying he believed all the laws and phenomena of the universe would be subdued by the mind of man, yet there seems to be no limit to man's progress in that direction.

CLASS ROOM SAYINGS.

"Tin is abstracted from its ores by heating with coke."

"Succulent roots are roots that stick to the walls by suction."

"Inertia is the state in which all inanimate objects stand and sometimes in which all animate objects stand."

"Ozone is obtained by subjecting oxygen to electrical shocks."

"Latent heat is heat that is left over."

"Energy of our bodies is traced back to horse power."

"A moment of force is a watt."

"Boiling point is the place where the saturated heat inside is equal to the heat outside a vessel."

How the world would be changed if there was no nitrogen:

War would never come to pass,
  And we wouldn't need to fear
That our friends and relatives,
  Had lost an eye or ear.

Gasoline accidents would be no more
  And explosions would be very rare,
And, last of all, we would have to breathe less,
  For there wouldn't be so much "air."

FALLING BODIES IN ANCIENT AND MODERN TIMES.

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Introduction

> Thrown by angry Jove,
> Sheer o'er the crystal battlements; from morn
> To noon he fell, from noon to dewy eve,
> A summer's day; and with the setting sun
> Dropt from the Zenith like a falling star
> On Lemnos, th' Aegean isle.

Thus Milton relates the fall from heaven of the erring angel. The poet's massive mind portrayed events on a gigantic scale. The curious minded reader may view the picture from many angles. If drawn to earth, the angel, though an incorporeal spirit, must have been subject to the force of gravity; in other words, he must have possessed weight. The question has been asked, with what velocity did the disgraced inhabitant of heaven fall upon the Aegean isle? The answer to this comes out differently according to whether we follow the teachings of Aristotle or of Galileo. Aristotle teaches that other things being equal, the velocity of fall depends upon the weight; an angel ten times heavier than another would fall ten times faster. This doctrine was the unfortunate speculation of a philosopher which remained unchecked by the facts of careful experiment. And people believed that a body ten times heavier would fall ten times faster, simply because the great Aristotle had said it. There were doubters, here and there, but their voices were not heard.

It required the genius and the power of appeal of Galileo to successfully direct attention to the falsity of Aristotle's doctrine. Galileo was then a young lecturer at the University of Pisa. For a mere youth to attack a doctrine set forth by the greatest philosopher of antiquity seemed presumptuous, but we are told that Galileo did this; that he appealed to experiment; that he experimented repeatedly, dropping different weights of various materials, wood, stones, lead, etc. It is said that the now celebrated leaning tower of Pisa offered fine opportunities for the trials, that at one time he experimented at this tower before the students and faculty of the university, causing a sensation. Some of the supporters of Aristotelianism resented these attacks upon their favorite doctrine. Perhaps they were jealous of Galileo and the attention which his brilliant work commanded. Their opposition was declared to be one of several factors which led to Galileo's leaving Pisa and accepting a position at the University of Padua.

Wreckers and Re-builders of History.

Recent criticism endeavors to sift the truth from a conglomerate mass of facts and fiction in the history of science. The movement is to be welcomed, whenever the work is carefully done. But an iconoclast may brush aside both facts and fiction, and leave unto the reader of science only a barren waste. The rebuilder of the historic structure may be an untrained architect with no sense of proportion or comprehension of the values of historic materials. The subject of falling bodies has had its trained historians and its amateurish historians. Comparatively simple questions have received different answers. Has Aristotle been misrepresented by Galileo to modern writers? Did Aristotle actually claim that a heavier body falls faster in proportion to its weight? Some years ago an English writer, J. H. Hardcastle, denied this. Quoting a brief passage from Aristotle he argued that historians had grievously erred, that the great Peripatetic had been stupidly misunderstood, that Aristotle had in mind what we now call "terminal velocity," at which the retardation of the medium (air) is exactly equal to the acceleration of gravity. Says Hardcastle,[1] "a penny can never fall faster than about 30 feet a second through air. I performed the experiment last week, dropping pennies from Clifton Bridge, 250 feet, into the Avon. They take eight or nine seconds to reach the water." Other of England's noted scientists made brief comments, basing their arguments on the one passage taken from Aristotle.[2] Thus G. Greenhill, William Ramsay, and Oliver Lodge arrive at the conclusion that Aristotle has been misinterpreted, that he did not mean a body ten times heavier would fall from rest ten times faster, that he was thinking of a body moving through a resisting medium, like the air, which greatly modifies the motion, and was considering, not its initial, but its terminal velocity. Greenhill interprets Aristotle as teaching "that the terminal velocity of a body in a medium is proportional to its weight," a law "justified by Newton in his experiments in St. Paul's," and explained in Newton's *Principia*, Book II, Proposition 40. Such terminal motion, says Greenhill, is exemplified in the motion of "a raindrop or hailstone falling vertically in the air, or of a smoke particle up the chimney." But Galileo, it was argued, discussed an altogether different question, namely, "the start of such a body from rest."

[1] *Nature*, London, Vol. 92, 1914, p. 584.
[2] *Nature*, Vol. 92, 1914, pp. 584, 585, 606.

Ramsay, the discoverer of argon, refers to the German chemist Ostwald as pointing out that "Aristotle was much more impressed with the retarding effect on the velocity of the mass of the medium through which the falling mass fell than with the law of 'free fall.'"

Oliver Lodge emphasized "the fact that 'terminal velocity' is the best instance of Newton's first law of motion in actual operation." These remarks on terminal velocities are exceedingly interesting and instructive. It is to be noted that Greenhill, Ramsay, and Lodge did not themselves go back to search in Aristotle's writings; their deductions were based on the one quotation (the first one given below) from Aristotle which had been brought to their attention. As a matter of fact, that one quotation gave only a partial exposition of Aristotle's theory. A study of all important passages on falling bodies in his *Physica* and his *De Coelo* (a half dozen passages or more) shows[3] that Galileo had correctly interpreted the great Stagirite.

The leading passages in Aristotle are as follows:

1. In his *Physica*, Book IV, cap. 8: "That which is heavier. . . . , other things being equal, moves faster through the same space, and indeed faster according to the ratio of the magnitudes of the things, so that this must happen also through a vacuum. But this is impossible; for why should it move faster? In a plenum this is necessarily true, because the larger moves more rapidly by its power of greater penetration."

2. In his *De Coelo*, Book I, cap. 6, we find: "If such and such a weight is moved so and so far in such and such a time, then some larger weight will be moved through the same distance in still shorter time, shorter in the inverse ratio of the weights. . . . A limited weight can pass over any limited line in a limited time."

3. Another part of the *De Coelo*, Book I, chap 8, contains the statement that "if there were an unlimited increase in the weight, there would be also an unlimited increase in velocity." The volume of a falling body is especially considered in the *De Coelo*, Book IV, chap. 1: "That body is heavier than another which, in an equal bulk, moves downward quicker."

4. In his *De Coleo*, Book III, chap. 2, he accepts the view of Anaxagoras that all bodies in the universe were originally at rest. Aristotle held that each body was light or heavy and had power to move.

"For suppose A without weight, but B possessing weight; and let A pass over a space CD, but B in the same time passes over a space CE —for that which has weight will be carried through the larger space. If now the heavy body be divided in the proportion that space CE bears to CD, . . . and if the whole is carried through the whole space CE, then it must be that a part in the same time would be carried through CD. Consequently the body without weight and the one possessing weight pass over the same distance, which is impossible."

[3]The present writer has briefly discussed this matter in *Science* N. S., Vol. 51, 1920, pp. 615, 616.

5 In *De Coleo*, Book IV, chap. 2, Aristotle argues, likewise, that the more fire will move upward with greater speed and the less fire with less speed, and similarly for the downward motion of more gold and more lead. His words are:

"but the *more* fire is moved upward faster than the little fire, and, on the other hand, downward faster the more gold and lead."

Here, as in most other places, Aristotle does not consider the shapes of the moving bodies. However, in *De Coleo*, Book IV, chap. 6, he does ask the question why a flat piece of brass or lead floats on water, while a needle sinks, but he does not arrive at an illuminating explanation.

These quotations show that Aristotle considered his law as applicable, to a body not only when moving through a resisting medium, but even when moving in a vacuum, if such existed. According to the teaching of Democritus and other atomists, the material universe was made up of moving atoms; it was thought that these atoms could not move, unless there was a vacuum for them to move in. Aristotle rejected the atomic theory and endeavored to demonstrate the non-existence of a vacuum. One of his proofs, as we have seen above, was that if his general law of falling bodies were applied to the vacuum, absurd conclusions would follow; hence, he argued, the vacuum cannot exist.

A close examination of the passages which we quoted shows that Aristotle considered his law applicable when the motion took place from rest as in (4), when there was no upper limit to the weight that the moving body may have as in (3), when the time of motion may be reduced or increased as in (2), and when the moving bodies are different weights of any metal, like gold or lead, as in (5). No restriction is placed by Aristotle to the combination of some or all of these four conditions in one and the same motion. As already pointed out, he was willing to apply his law even to motion in a vacuum (were a vacuum possible) as in (1). It appears, therefore, that Aristotle allowed his law, that the velocity of a falling body is proportional to its weight, a generality of application which certainly did include the special conditions, under which Galileo performed such experiments as dropping of a small shot and a large shot through the air from the leaning tower of Pisa; Galileo interpreted Aristotle correctly.

THE TOWER OF PISA.

"The truth should now be better understood;
Old things have been unsettled." —[Wordsworth.

The light of critical historical research was directed next upon Galileo himself; did he actually perform experiments at the leaning tower of Pisa, or are the accounts as given by Galileo's pupil and biographer, Vincenzio Viviani, and found in our histories of physics, mythical? If Galileo did experiment, what was the exact nature of his tests? One of the first doubting Thomases was a German scientist, Dr. Emil Wohlwill[4] of Hamburg. On the hypothesis that Viviani's account contained much that was mere hearsay, he tried, in 1905 and 1906, to reconstruct the history of Galileo's research on falling bodies during the three years when he was at the University of Pisa, by using sources independent of Viviani's account. As a basis for discussion, we begin by quoting Viviani's account of experiments[5] at the leaning tower:

"At this time he became convinced that in investigations of phenomena of nature there is demanded a true knowledge of the nature of motion, as expressed in that philosophical and common axiom Ignorato motu ignoratur natura; he devoted himself wholly to the contemplation of this subject; and then, to the great consternation of all philosophers, he showed by means of experiments and by crucial proofs and explanations, that numerous declarations of Aristotle on motion were false, which up to that time had been considered perfectly clear and not subject to doubt. Thus he demonstrated among other things that the velocities of bodies of the same material but of different weights, when they move through the same medium, are not as Aristotle claims in proportion to their absolute weights, but on the contrary all move with equal velocity. This he proved by repeated experiments made from the height of the Campanile of Pisa in the presence of the other professors and philosophers and the whole student body, and also that the velocities of one and the same body in different media are not inversely proportional to the resistance or density of the media, which manifest absurdity in contradiction to experience would follow from the same assumptions (all of which one finds treated by him more fully in his Dialogues of the Two New Sciences).

The part placed in brackets () is historically important; it is given in the first of two manuscript copies of the life of Galileo, written by Viviani and published by Salvino Salvini in 1717; it is crossed out in the second or revised copy. For the reason that the revised copy is considered more authoritative, this part is omitted by A. Favaro, from the regular text, in the recent National Edition of Galileo's work, and is given only in a footnote.

The above account impresses Wohlwill as legendary. Rejecting it and looking around for other sources of information, he is startled by the fact that contemporary writers make no mention of the public experiments of Galileo at the leaning tower. Not

[4]Emil Wohlwill, "Galilei-Studien," Mitteilungen zur Geschichte der Medizin und Naturwissenschaften, 1905, Vol. IV, pp. 229-249; Vol. V, 1906, pp. 230-249, 439-464.

[5]Le Opere di Galileo, Galilei. Edizione Nationale, Vol. XIX, Firenze, 1907, p. 606.

even Galileo himself refers specifically to the leaning tower of Pisa in any of his writings, although a half a dozen times he speaks in his *De Motu* of experiments on bodies falling from a tower—"ex turri," "ex alta turri," "ab alta turri."[6]

Wohlwill examined Jacopo Mazzoni's book entitled, *De comparatione Aristotelis et Platonis*, published only five years after Galileo left Pisa (1591). Mazzoni[7] was a colleague and friend of Galileo, was professor of Aristotelian Philosophy at Pisa, and had begun to doubt the infallibility of Aristotle; nevertheless, he makes no reference to any Galilean experiments on falling bodies at Pisa. Likewise silent is Giorgio Coresio,[8] a lecturer on the Greek language, who in a publication appearing 21 years after Galileo left Pisa opposed Galileo's and Mazzoni's views on motion but made no mention of experiments on falling bodies due to Galileo at Pisa. In a reply to Coresio, prepared by Galileo's pupil, Castelli,[9] nothing is said relating to his master's Pisan experiments. A similar silence is observed by another pupil of Galileo, Vincenzio Renieri, who about 45 years after Galileo left Pisa described some experiments of his own, performed at the leaning tower. Such omissions are indeed surprising, but are not at all unusual in affairs relating to men who became great.

Thus far the argument against Viviani is purely negative and amounts to this: Viviani's life of Galileo is mythical regarding experiments on falling bodies performed at the leaning tower, because four other men who did not write lives of Galileo wrote tracts or letters touching on falling bodies without mentioning Galileo's Pisan experiments. Wohlwill frankly admits the weakness of this part of the argument.

Wohlwill proceeds next to present positive evidence in favor of his thesis that Galileo, while at Pisa, did not perform the public experiments attributed to him by Viviani. Galileo wrote a long tract on motion, called *De motu*, which was not published in his lifetime, and was first printed in full by Boncompagni in his *Bullettino* about half a century ago and again by A. Favaro in the first volume of the National Edition of Galileo's Works, in 1890. Wohlwill considers this *De motu* the most reliable source of information on Galileo's experiments and cogitations during the years 1589-1592. The tract is a decidedly immature pro-

[6] Edizione Nazionale, Vol. I, pp. 263, 273, 334, 406, 407.
[7] See Edizione Nazionale, Vol. II, pp. 193-202.
[8] See Edizione Nazionale, Vol. IV, p. 203.
[9] Edizione Nazionale, Vol. IV, p. 245.

duction as compared with the publications which Galileo allowed to be printed during his lifetime.

The following passage in his *De motu* indicates that, like Kepler, Galileo, in his younger years, fretted under the restraint imposed upon his theoretical speculations by the stern facts of experience. Once he said in his *De motu:* "We will operate more with causes than with examples (for what we seek are the causes of phenomena which experiment does not reveal)."[11]

In another place in this voluminous tract he discredits the value of experiment, saying: "It is of no consequence, if occasionally experience contradicts the well-grounded theory."[10]

In fact, the larger part of his *De motu* is made up of argumentation, rather than the accounts of experiment. Yet, a closer examination shows that even in this early document experience is nowhere rejected, but is clearly accepted as the court of last appeal.

There are indications that some of his early experiments were crudely performed. As ill fortune would have it, the cruder experiments related to the most subtle part of his speculations. He was at one time impressed by the views of the great twelfth century Spanish-Arabic philosopher, Averroës, who held that in a vacuum the lighter body fell the faster. At such a point, speculation was rampant, because no direct appeal to experiment was possible, except for deductions drawn relating to the fall through the air. The following passage hardly sounds like the Galileo of later years:[12]

"They (followers of Averroës) say that air in its own region is heavy; that thence it follows that those objects which have more of air in airspace are heavier (and this is also the opinion of Aristotle): from which it follows, since a wooden sphere, for example, has more air in it than a leaden one, as it has three gravitating elements (gravantia), namely air, water, and earth; lead indeed having within itself a modicum of air, has as it were as much as two gravitating elements. Hence it follows that a wooden sphere falls faster than one of lead. . . . Experiment, however, shows the contrary, for it is a fact that at the very start wood falls faster than lead; but somewhat later the lead accelerates its motion and leaves it behind and if dropped from a high tower, surpasses it by a long distance; and this experiment I have made often. It behooves us to set up a surer cause upon a firmer hypothesis. Oh, how easily could true declarations be extracted from true principles. . . . But in this case a great difficulty arises which should not be disguised by silence."

We have quoted above the passages which exhibit young Galileo scientifically at the greatest disadvantage. But they are interesting as portraying a stage in the evolution of a great

[10]Edizione Nazionale, Vol. I, 1890, p. 263.
[11]Edizione Nazionale, Vol. I, p. 406.
[12]Edizione Nazionale, Vol. I, pp. 333, 334.

scientific mind which had been nursed in Aristotelian philosophy.

In part an appeal to intuition and in part perhaps an appeal to experiment is contained in the following passage from the *De motu*

"Who would ever believe that if, for example, from the orb of the moon two leaden balls were dropped, one a hundred times greater than the other, and the greater reached the earth after an hour, the smaller would require one hundred hours for this motion? Or, if from a high tower two stones were thrown down the same moment, one twice as great as the other, that when the smaller is at the middle of the tower the larger has already reached the earth?"[13]

A clearer appeal to experiment is found in the following passage which Wohlwill did not quote:[14]

"For we see from experience that of two balls of equal volume, one twice as heavy as the other, dropped from a tower, the heavier does not reach the ground twice as fast."

A similar statement of an experiment "ex turri" is made again later.[15]

In considering the ratios of velocities in different media with the ratios of weights of the moving bodies, his reasoning is pinned down to the following experimental data:[16]

"This indeed must be borne in mind, if one happens to consider two balls in air, one of iron, the other of wood, and finds the iron one to be ten times heavier than the wooden and then lets both down into the water, the iron one descends, but the wooden one not in the least; and consequently, as regards motion, the iron ball bears no ratio to the wooden one."

From these quotations it is evident that Galileo experimented on falling bodies, probably while he was in Pisa and probably at the leaning tower. Wohlwill admits this, but claims that Galileo's *De motu* proves that while in Pisa, Galileo had not yet reached clear, accurate and convincing results on falling bodies and therefore could not possibly have performed the public experiments and could not have boldly challenged the teachings of Aristotle, before a University assembly, in the dramatic manner set forth by the biographer Viviani. Hence, claims Wohlwill, Viviani's account must be based merely on gossip or hearsay and must be rejected absolutely.

Wohlwill's conclusion rests on two vicious assumptions: (1) That the *De motu* was written toward the close of Galileo's activity at Pisa, about 1592; (2) That the *De motu* reveals an uncertain state of mind on the part of Galileo, while he was residing at Pisa, that is incompatible with the aggressive, chal-

[13]Edizione Nazionale, Vol. I, 1890, p. 263.
[14]Edizione Nazionale, Vol. I, p. 267.
[15]Edizione Nazionale, Vol. I, p. 273.
[16]Edizione Nazionale, Vol. I, 1890, p. 267.

lenging attitude which alone could lead to the public experimentation that Viviani attributes to him at the leaning tower.

The first assumptin is insecure, since no one has been able definitely to fix the exact time when Galileo wrote the *De motu;* Galileo's biographer, G. B. C. de Nelli, inclines to 1590 as the date.[17] It is not absolutely certain that Viviani ever saw the *De motu.* In 1674 he makes mention of a Galilean manuscript, entitled *De Motu Antiquiora,* the content of which gives evidence, Viviani says, of being among the first studies of young Galileo.[18] Perhaps this document was the *De motu* which has come down to our time.

Wohlwill's second assumption is far from established. What perplexed young Galileo most was motion in a vacuum; Viviani makes no mention of Galileo's speculations on this point. Viviani refers to the dropping of weights through the air from the leaning tower, and to the motion of one and the same body through different media. Our quotations from the *De motu* show that, except the single case of the motion of wood and lead during the first moments of their fall from rest, his conclusions relating to bodies falling through air were clear cut, definite and accurate. Doubts as to the vacuum need not prevent Galileo from attacking Aristotle on the theory of bodies falling *through the air.* Galileo's attitude toward Aristotle as set forth in the *De motu* is a vital question in our investigation which is not adequately treated by Wholwill. He could have conveyed to the reader a truer picture had he quoted some of Galileo's chapter headings in the *De motu,* such as,

"Where it is proved contrary to Aristotle, that if there is a vacuum, motion in it does not become instantaneous, but is finite."

Altogether there are ten chapters[19] which, in their very headings, indicate Galileo's strong opposition to Stagirite teachings; such phrases as "contra Aristotelem," "in quo error Aristotelis manifestatur" are of constant occurrence.

If still further evidence is desired, consider passages in the *De motu* like the following:[20]

"In this treatise we follow the method, that the things to be proved are made to depend upon what has been already established. At no time indeed will I set down as true what must first be demonstrated. This method the mathematicians have taught me. It is not sufficiently heeded by certain philosophers who, more often, in teaching the elements of physics, substitute in its place things handed down from antiquity in the

[17]Edizione Nazionale, Vol. I, p. 245.
[18]Quinto libro degli elementi d'Euclide . . . da Vincenzio Viviani, 1674, pp. 104, 105; also Edizione Nazionale, Vol. I, p. 246.
[19]Edizione Nazionale, Vo. I, pp. 262, 276, 285, 289, 294, 302, 315, 323, 328, 355.
[20]Edizione Nazionale, Vol. I, p. 285.

books *De anima*, or *De Coelo*, or even in the books on Metaphysics. Nor indeed is this all, for even when teaching logic itself, they constantly have on their lips only those things which are handed down in the books of Aristotle. Accordingly, when teaching their pupils, they expect them to know things and derive their doctrine not from actual investigation, but from what is in fact unknown and mere hearsay. By such teaching, moreover, they never know the reasons of things, but take things for granted simply because Aristotle says so."

These are the words of a man who felt intensely and would not hesitate to appear openly before a university assembly to demonstrate the errors of Aristotle. If we consider, moreover, that the public experiments mentioned by Viviani touched points on which Galileo had reached clear views as early as the time when he wrote his *De motu*, it follows that Wohlwill's contention, that Viviani's account of Galileo's public experiments on falling bodies at the leaning tower are a myth, is wholly unsupported by the evidence. This stands out still more clearly when we remember that there is no valid objection to the assumption that Galileo wrote the *De motu* during the *early part* of his lecturing period at the University of Pisa and gave the public exhibition described by Viviani toward the *end* of that period.

As regards the nature of the experiments performed at the leaning tower, it is clear that we know their purpose, which was to ascertain whether heavy bodies fall through the air faster than lighter bodies. In his *De motu* he experimented with stones, cannon balls, lead, wood. In his *Dialogues Concerning Two New Sciences*,[21] 1638, Galileo says:

"But I, Simplicio, who have made the test, can assure you that a cannon ball weighing one or two hundred pounds, or even more, will not reach the ground by as much as a span ahead of a musket ball weighing only half a pound, provided both are dropped from a height of 200 cubits."

This quotation, taken in conjunction with the last part of the sentence which we quoted from Viviani's biography, viz.: "all of which one finds treated by him more fully in his *Dialogues of the Two New Sciences*," has led historians of physics to state that at the university convocation Galileo dropped from the leaning tower a one pound shot and a one hundred pound shot. That these were the particular weights dropped appears to us less certain now than it did formerly. Since the publication of the National Edition of Galileo's works, under the editorship of Antonio Favaro, we know that Viviani, when he revised the first draft of his biography, crossed out the part of the sentence just quoted. However, we know the essential nature of Galileo's experiments, namely, the dropping of different weights of solids.

[21] Translation by H. Crew and A. De Salvio, New York, 1914, pp. 62, 65, "First Day."

Viviani does not speak simply of one experiment, but of "repeated experiments." Historical criticism has, therefore, not robbed us of the picture of Galileo, young, enthusiastic and imprudent, experimenting at the tower of Pisa and before a university assembly disproving the statements of Aristotle on bodies falling through the air.

It is gratifying that our conclusion based on the examination of the *De motu* can be accepted as valid without having to do violence to the testimony of Galileo's most devoted pupil. Viviani was with his master much of the time during the last three years of Galileo's life, became himself a noted scientist, and was respected for his probity and uprightness. If conflict really existed between his testimony and that of others, then his testimony could not be lightly brushed aside. But there is no conflicting evidence. The historic facts fit one to the other harmoniously. It is strange that a man of the acuteness of the German critic should have failed to see the historic data in their true light.

That Wohlwill's conclusions are vulnerable is illustrated also by another incident. Viviani states that the Swedish King, Gustavus Adolphus, when a youth, traveled *incognito* in Italy and, at Padua, attended the lectures of Galileo. On the general contention that Adolphus never was in Italy, Wohlwill quotes this as another illustration of Viviani's inaccuracy. Such negative assertions are not only hard to prove, but dangerous. In this case, Antonio Favaro, the great student of Galileo, was able to quote evidence from a number of contemporary writers to show that Adolphus really was in Italy and that Viviani was right.[22] After a lifetime given to the study of Galileo and his contemporaries, Favaro is able to state that Viviani wrote a biography of Galileo, too brief to be sure, but in the main remarkably accurate and trustworthy.[23]

The need of a critical examination of Wohlwill's conclusions pertaining to Galileo at Pisa is the more evident from the fact that the most recent History of Physics, namely that by E. Gerland,[24] acdepted Wohlwill's conclusions bodily.

[22] A. Favaro in Atti del Reale Istituto Veneto di Scienze, Lettere ed Arti Anno, 1905-1906, Tonio 65 Parte seconda, pp. 17-39.
[23] A. Favaro, *Amici e Corrispondenti di Galileo Galilei*, XXIX Vincenzio Viviani, Venizia, 1912
[24] Dr. E. Gerland, *Geschichte der Physik durchgesehen von Dr. H. V. Steinwehr*, München und Berlin, 1913.

GERBERT'S LETTER TO ADELBOLD.

By G. A. Miller,
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One of the most astounding comedies of errors in the history of elementary mathematics relates to Gerbert's letter to Adelbold, as may be inferred from the brief articles by the present writer published in the May, 1921, number of the *Scientific Monthly* and in the June, 1921, number of the *American Mathematical Monthly*. Neither of these two articles furnishes the full data upon which the conclusions were based and hence they serve mainly to direct the attention of the careful reader to a few sources of interesting information and to present to the indifferent reader statements which he may accept or reject as his confidence, or lack of confidence, in the judgment of the present writer on such questions may dictate.

Teachers of mathematics should be especially interested in Gerbert, who died in 1003 as Pope Sylvester II, since he is the only man who rose to the position of Pope of the Catholic Church from that of being the most influential teacher of mathematics and other subjects in his generation. The mathematics which Gerbert taught would now be regarded as very elementary, and hence he should be classed with the teachers of secondary mathematics rather than with those of college grade. He lived at a time when even the leading mathematicians of the world used little of their mathematical heritage and added practically nothing thereto for the good of future generations.

Statements found in various well known modern histories of mathematics might lead one to infer that the letter under consideration had been a real contribution towards the increase of mathematical knowledge. Among these statements are the following: It is "the first mathematical paper of the Middle Ages which deserves this name." It explains "the reason why the area of a triangle, obtained by taking the product of the base by half its altitude, differs from the area calculated according to the formula $\frac{1}{2}\, a\,(a+1)$." It "gives the correct explanation that in the latter formula all the small squares, into which the triangle is supposed to be divided, are counted in wholly, even though parts of them project beyond it." To provide a solid basis for further observations and on account of the intrinsic value of this letter for teachers we quote in full the extant part thereof, as follows[1]:

"In these geometric figures which you have taken from us

[1]Gerberti, *Opera Mathematica*, by N. Bubnov, 1899, p. 43. Professors H. J. Barton and W. A. Oldfather assisted me in the translation of this letter.

there was an equilateral triangle whose side was 30 feet, altitude 26, and, according to multiplication of side and altitude, the area 390. If you measure the same triangle without paying attention to the altitude, viz., that one side be multiplied into itself and to this product the numerical measure of one side be added and from this sum half be taken, the area is 465. Do you not see how these two rules disagree? But also the geometric rule, which, by taking into consideration the altitude makes the area 390 feet, has been minutely discussed by me, and I grant that its altitude is only $25^{5}/_{7}$ feet and its area $385^{5}/_{7}$ feet. And let this be a universal rule for you for finding the altitude in every equilateral triangle; from the side always take away a seventh and assign the six remaining parts to the altitude."

"To make what has been said more intelligible permit me to exemplify with smaller numbers. I give you a triangle whose side is 7 feet long. This by the geometric rule I measure thus. Take away a seventh of the side and the six sevenths which are left I give to the altitude. I multiply the side by this and say 6 times 7, which gives 42, the half of this, 21, is the area of said triangle. If you measure the same triangle by the arithmetic rule and say: 7 times 7, that makes 49, and you add the side, making 56, and divide so that you may find the area, you will obtain 28. Behold thus in a triangle of one magnitude there are different areas, which is impossible."

"But that you may not wonder longer I shall explain to you the cause of the diversity. I believe it is known to you what feet are said to be linear, what square, and what cubic, and that in measuring areas we take only square feet. However small a part of these the triangle touches the arithmetic rule computes them as integral. Allow me to give a diagram[2] that what is said may be more clear."

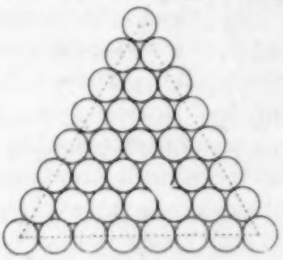
"Behold in this little diagram there are included 28 feet, not all of which are integral. Whence the arithmetic rule, taking the part for the whole, counts the halves with the integers. But the skill of the geometric discipline throwing away the small parts extending beyond the sides and putting together the remaining halves cut in two on the inside of the lines considers that only which is enclosed by the lines. For in this little diagram of which the sides are each 7 feet long if you seek the altitude it is 6 feet. Multiplying this number 6 by 7 you complete, as it were, a rectangle of which the front is 6 feet, the side 7 and the area of it you thus determine to be 42 feet. If you take half

[2]Cf. Fig. 2.

of this you leave a triangle of 21 feet. To understand more clearly use your eyes and always remember me."

It may be assumed that all modern teachers of mathematics agree that if any of their students would present the vague arguments found in this letter to explain why the formula $\frac{1}{2}\, a\,(a+1)$ does not give the same result for the area of an equilateral triangle of side a as the formula $(a/2)^2\sqrt{3}$, which is equivalent to the product of one-half the base into the altitude, they would reply that these arguments failed to explain the difference in question. Hence the statement that Gerbert gave a *correct* explanation of this difference, which appears in various well-known modern histories of mathematics, including that of M. Cantor, volume I (1907), page 865, is inaccurate.

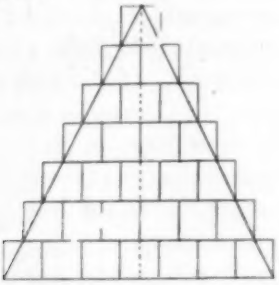
It may be of interest to consider here briefly a possible explanation of the fact that the Roman surveyors assumed that the formula $\frac{1}{2}\, a\,(a+1)$ represents the area of an equilateral triangle whose side is a. The ancient Greeks used the term *triangular numbers* for the positive integers of the form $\frac{1}{2}\, a\,(a+1)$. From the facts that the number of equal tangent circles arranged



as in the adjoining figure is of this form and that the centers of the outside circles in this figure determine an equilateral triangle, it seems natural to assume that the term triangular numbers originated in connection with some such figure.

If, in the given figure, the number of circles is increased indefinitely without changing the centers of the corner circles, the number of these equal tangent circles will always be expressed by the formula $\frac{1}{2}\, a\,(a+1)$ where a represents the number of those circles whose centers determine a side of the given equilateral triangle. As the lower limit of each of these circles is a point it would appear that the number of points in the surface bounded by the given equilateral triangle should be $\frac{1}{2}\, a\,(a+1)$, where a represents the number of points in a side. On the other hand, if the area of the given triangle is expressed in smaller and smaller square units, such that the length of the

base is equal to a of the corresponding linear units, the area of this triangle is always $(a/2)^2\sqrt{3}$. In fact, $\frac{1}{2}a(a+1)$ has for its limit as a is indefinitely increased the area of an isosceles triangle whose base is equal to the altitude as may be seen directly by means of figure below.



It would thus seem that the assumption that a surface is composed of points which are the limits of equal tangent circles leads to a contradiction. At any rate, if our hypothesis in regard to the origin of the formula $\frac{1}{2}\ a\ (a+1)$ for the area of an equilateral triangle whose side is a is correct, it is interesting to note the dilemma into which the ancient Roman surveyors were led by assuming that a surface is made up of points which are the limits of circles. While these surveyors were not good mathematicians they had sufficient mathematical instinct to recognize that if two assumptions lead to contradictory results they cannot both be true—an instinct which is sometimes disregarded by eminent modern mathematicians.

Returning to Gerbert's letter it may first be noted that the rule for finding the altitude of an equilateral triangle which is found therein is inaccurate since this altitude is equal to a side multiplied by $\frac{1}{2}\sqrt{3} = .8660+$, while $6/7 = .8571+$. It is true that the latter is a fairly close approximation but the language of the letter implies that the writer does not realize that his rule gives a result which is only approximately correct.

The main claims for the mathematical merits of the letter in question relate to the so-called explanation of the reasons why the "geometric rule" and the "arithmetic rule" give different results. Judging from his language the diagram to which Gerbert refers may have been like the adjoining figure which he probably assumed to represent an equilateral triangle. At any rate, if the number of squares on the base line had been 30, in accord with the first triangle to which Gerbert refers in this letter, the vertex of the equilateral triangle constructed on this

base would have been below several of the topmost squares, and hence a number of the squares of the corresponding figure would not have been touched by this triangle, lying entirely outside of it.

The method of counting as wholes also those squares which lie only partly within the triangle would therefore not have yielded the number 465, required by the formula $\frac{1}{2}\, a\, (a+1)$. Even for the special case when $a = 7$ the explanation is too vague to be called correct. Hence it seems clear that the statement that Gerbert gives a *correct* explanation[3] of the fact that different results are obtained by using two different methods for finding the area of an equilateral triangle is very misleading.

The claim that the letter in question is "the first mathematical paper of the Middle Ages which deserves this name" is even more misleading. It is true that not much mathematical progress was made from the beginning of the Middle Ages to the time of Gerbert, but various papers which were produced during this period are much more meritorious than this letter by Gerbert. In support of this assertion it is only necessary to recall that much of the work of the Hindu and the Arabian mathematicians belongs to this period. In particular, the work from which our term for algebra is derived appeared therein, as well as the dissertation on amicable numbers by Tabit ibn Korra, which has been called "the first known specimen of original work in mathematics on Arabian soil."[4]

Teachers of mathematics should be interested in the letter of Gerbert quoted above not so much on account of the many historical errors relating thereto as on account of the fact that it exhibits a type of the best mathematical thinking among the Romans during the tenth century. From this standpoint the letter is very instructive and presents to us a picture of weak mathematical thinking on the part of a gifted man. We are thus reminded of the need of carefully graded mathematical aids for our students in order that they may reach even at an early age a mathematical maturity which excels that of the most gifted among the Romans who had to find their way without such aids. Gerbert's letter can profitably be read repeatedly by teachers of mathematics, since it illustrates a type of mathematical thinking which is not only interesting historically but is represented by many students at the present time.

[3] This claim seems to have been made first by H. Hankel, *Zur Geschichte der Mathematik*, 1874, p. 314.
[4] F. Cajori, *History of Mathematics*, 1919, p. 104.

THE MATHEMATICS NEEDED IN FRESHMAN CHEMISTRY.

By Lewis W. Williams,

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This study was made to throw some light upon the criticism that high school students entering college with a year of algebra and a year of geometry do not have sufficient mathematics to handle Freshman chemistry. With this practical purpose in mind the book selected for examination was the text, "A Textbook in Chemistry," by Professor W. A. Noyes, required in the Freshman chemistry course in the University of Illinois.

Because the text consists of two distinct phases—exposition of the principles of chemistry and problems given for solution—the study is divided into two parts and these are treated independently. In the analysis of the expository portion a careful examination of the text was made to determine what words or expressions were used which were distinctly mathematical or which implied mathematics. All others were omitted. These were then classified and their frequency determined. In the second part of the study the problems were solved and where more than one solution was possible, the simplest was selected. The operations and quantities were classified as in the first part and the frequency determined.

It was early seen that some standards had to be set up for determining whether or not a word or expression should be included in the study or be omitted. The following principles were chosen for this purpose:

1. Expressions giving dates, either specific or general, imply mathematics. Example 1744, last century.
2. Denominate numbers are distinctly mathematics. Example 5° F., 76 cm.
3. Expressions having several meanings or uses are distinctly mathematical if, from the context, a mathematical meaning is evident; if not, they are omitted. Example, equivalent, constant.
4. Expressions having no use or meaning other than in a mathematical sense are distinctly mathematical. Example, volume, number.
5. Expressions not classed under Principle 4, but involving a use of mathematics, are considered as implying mathematics. Example, rate of expansion, equilibrium, divisions on a scale.
6. Cardinal, ordinal and numeral adjectives are distinctly mathematical. Example, three, first.
7. Equations, formulas, symbols, etc., are distinctly mathematical. Example, AB = 180, $\frac{F-32}{180} = \frac{C}{100} = \frac{R}{80}$

Words or expressions which could not be classed under the above principles were omitted.

Exposition.

Table I shows the mathematical concepts, a reading knowledge of which is necessary for an understanding of the expository portion of the text. It is obvious from an examination of the list that the concepts most commonly used are those most commonly found in arithmetic or in the first year high school algebra, or geometry. And those less frequently used are either found in the same sources or are not found in second year algebra. As a matter of fact, it would appear that if the less familiar terms, as rhombic hexahedron, are to be taught as a prerequisite to chemistry, it would be less serviceable to teach advanced mathematics than to give specific instruction in the meaning of this list.

Table II indicates the use of many common terms, as cents, miles, years, and two other groups of terms. These are the metric system, (32 terms), and certain terms in physics, as

TABLE I.

Mathematical Concepts Used in Exposition.

add	2	formulas	1
added	2	fraction	16
absolute zero	3	fractions	2
abscissa	3	geometrical	1
adding algebraically	1	hemispherical	1
algebraic sum	1	hemihedral	1
amounts	1	hexagonal	1
angle	4	hexagonal pyramid	1
angles	1	hyperbola	1
apex	1	invariable	1
average	8	inversely	4
axis	13	inverse proportion	1
British Thermal Units	3	inversely proportional	1
calculate	1	last	28
centimeter	1	length	5
constant proportion	6	lengths	4
constant volume	1	line	2
constant	1	lines	4
cube	4	monoclinic	2
cubes	2	multiple	2
curve	1	multiples	2
curves	4	multiplied	2
cross section	1	multiplying	1
denominator	2	multiple proportion	3
diameter	8	negative	4
difference	5	number	64
direct proportion	1	numbers	4
divided	5	numerator	5
division	1	octohedra	6
dividing	1	octohedron	2
empirical	1	origin	1
equation	43	ordinates	3
equations	14	parallel	6
face	2	per cent	310
faces	2	percentage	8

perpendicular	1	six-sided	1
plane	5	square	3
plotted	1	square prism	1
positive	5	square root	3
prism	1	straight line	1
product	4	subdivide	1
proportion	51	subdivisions	1
proportional	14	submultiple	1
proportionably	1	subtract	1
pyramid	1	subtracted	2
pyramids	2	subtracting	2
rate	29	sum	5
ratio	24	surface	1
ratios	3	tetragonal	1
reciprocal	3	tetragonal pyramid	1
rectangular	1	tetrahedron	2
rectangular prism	1	triclinic	1
rhombic	5	triple	1
rhombic dodecahedron	1	unit	3
rhombic hexahedron	1	unity	1
rhombic prism	2	varies	3
rhombohedra	1	vary directly	3
right angles	4	vary inversely	2
right angled prism	1	velocity	34
series	12	volume	208
single	1	volumes	41
simple ratio	6	zero	1

TABLE II.

Denominate Numbers Used in Exposition.

amperes	4	kilogram	21
calorie	29	kilograms	5
centimeter	74	kg	1
cento	3	kilometer	1
cents	3	kilometers	2
century	2	kilos	2
centuries	5	kilowatt	1
c. c.	41	kilogram meters	5
cubic centimeter	22	liter	60
cubic decimeter	1	liters	45
cubic feet	6	meter	10
cubic millimeter	2	meters	12
day	2	metric ton	1
days	5	micron	1
decimeter	1	mile	1
degree	8	miles	1
°	865	milligram	22
$	8	milligrams	29
erg	5	milliliter	1
ergs	1	mm	104
feet	4	millimeter	3
footpounds	3	minute	14
gallon	1	minutes	3
gram	169	mouths	5
grams	172	ohm	2
g	12	ohms	1
half dozen	1	ounces	4
hours	7	pound	12
horsepower	2	pounds	4
inch	2	lb	1
joule	5	second	1
joules	7	square centimeter	2

sqcm	2	watt	4
square meter	1	weeks	5
ton	13	year	3
tons	11	years	45
volt	4		

TABLE III.

Other Terms Used in Exposition.

first	115	twice	20
second	54	double	4
1st	1	doubled	2
2nd	1	triple	3
once	1		

TABLE IV.

Common Fractions—in Figures and Words.

Common Fractions—in Figures		*and Common Fractions.*	
1/2	12	1/15	1
one half	35	one fourteenth	1
one & one half	2	1/20	1
two & one half	2	one twentieth	2
3½	1	71/29	1
fourteen & one half	1	hundredths	6
third	23	three one-hundredths	1
two thirds	6	1/120	1
fourth	17	one six-hundredth	1
three fourths	6	thousandth	3
¾	1	1/1000	4
20/4	1	one seventeenth-hundredth	1
one & one fourth	1	one millionth	3
one fifth	32	ten millionth	1
two fifths	1	1/3000000	1
three fifths	1	1/7000	1
four fifths	6	1/500000	1
3/5	1	1/5000	1
sixth	8	1/272	3
five sixths	1	1/272.8	1
seventh	7	1/272.9	1
one eighth	1	1/272.6	3
seven eighths	1	1/273	2
ninth	2	1/3.8021	1
5/9	1	4/3.1416	1
tenth	23	18/24	1
tenths	2	22/24	1
1/10	2		

amperes, calories, and watts (13 terms). Here, again, it is apparent that advanced mathematics is less essential than an intensive drill upon the meanings of the common concepts and upon the metric system, together with specific explanation of the thirteen physical terms, in case physics has not been studied previously.

Table III presents a list of very simple terms quite understandable by even those who have not studied airthmetic.

The significance of Table IV is apparent. The student needs a reading knowledge of fractions of all-sized denominators, and, although fractions with denominators larger than ten constitute only eighteen per cent of the total, they are of sufficient importance to require attention. They involve, also, a knowledge of

decimals, though not so extensive a knowledge as is required in later tables. Here, again, it is obvious that an intensive review of fractions would be more efficient than an advanced course in mathematics where the more difficult of these terms, which presumably give more trouble, would be encountered by chance, if at all.

TABLE V.

Decimal Fractions Used in Exposition.

Fraction			Frequency
.1	—	.9	33
.01	—	.99	140
.001	—	.998	50
.102	—	.996	75
.0002	—	.9996	49
.00002	—	.99999	66
.000008	—	.999982	14
.0000001	—	.0000026	6
.00000326	—		1

Table V clearly shows that a student needs to understand decimal notation to quite extended limits, although, as might be supposed, the lower places are most frequently used; four-place decimals, or less than four, constitute eighty-two per cent of the cases.

Table VI indicates that there were found 121 cases of mixed decimals with one place in the integral portion of the number and one in the fractional portion; as 4.3; and that 256 cases of the type 27.3, with two integral and one decimal place, were obtained, and so forth.

Table V contains the pure decimal fractions while Table VI contains the mixed decimals. The contrast in the use of common fractions and decimals is very evident when we note that fifty different common fractions were used and 920 different decimals. Were the frequency of the different fractions considered, this contrast would be much more emphatic. Pure decimals range from one place to eight places, the large majority being two, three, four and five places. Among the mixed decimals we find the following limits: From one unit place and one decimal place to four unit places and one decimal place, by far the greater number consisting of two or three unit places and one decimal place; from one unit place and two decimal places to three unit places to three unit places and two decimal places, most of them consisting of one or two unit places and two decimal places; from one unit place and three decimal places to three unit places and three decimal places, most of them consisting of two or three unit places and three decimal places. There were very few mixed decimals with four or five

decimal places. None were found with a higher number of decimal places than five.

TABLE VI.
Mixed Decimals Used in Exposition.

Place Integers	Place Decimals	Frequency
1	1	121
2	1	256
3	1	154
4	1	1
1	2	173
2	2	186
3	2	44
1	3	34
2	3	3
3	3	2
1	4	41
2	4	1
1	5	3
2	5	1

In Table VII, we find the integers used in the first part of the study. One place integers to the number of 570 were found in figures and 975 in words. They range from one unit place to twenty. There are very few beyond six unit places, however. Furthermore, most of the numbers consisting of more than four unit places end in one or more ciphers, which naturally simplifies their use.

Problems.

In Part II, all problems given for solution were solved. Each solution was then analyzed into its processes. In Tables VIII and IX, we have the processes and their frequency as found in the sixty-six problems. The striking thing about this table is the low frequency of equations, not only for two unknowns but also for one. This is in part explained by the fact that in a typical chemical problem, such as—How many grams of magnesium should be mixed with three grams of sand to form silicon?—the operation is not considered as algebraic but rather as the solution of an arithmetical proportion in which three terms are given to find the fourth.

2 Mg plus Si O_2 2 MgO plus Si
48.6 60.3 96.6
60.3 : 3 : : 48.6 : ? $48.6 \cdot 3 = 2.42$ Ans.
60.3

Certainly no algebra is needed for the solution of such problems. This brings us to the use of ratio and proportion.

TABLE VII.

Integers Used.

Place	Frequency Figures	Words	Total
1	570	975	1545
2	1150	39	1189
3	893	13	902
4	382	9	391
5	40	0	40
6	25	6	31
7	2	4	6
8	9	0	9
9	1	1	2
10	2	1	3
14	0	1	1
20	1	0	1

TABLE VIII.

Processes Used.

Equation 1st Degree, 1 unknown	7
Equation 1st Degree, 2 unknowns	2
Equation 1st Degree, 3 unknowns	0
Equation 2nd Degree, 1 unknown	0
Equation 2nd Degree, 2 unknowns	0
Arithmetical addition (integers)	72
Arithmetical subtraction (integers)	11
Arithmetical multiplication (integers)	149
Arithmetical division (integers)	25
Arithmetical addition (with decimals)	41
Arithmetical subtraction (with decimals)	1
Arithmetical multiplication (with decimals)	118
Arithmetical division (with decimals)	88
Arithmetical addition (common fractions)	0
Arithmetical subtraction (common fractions)	0
Arithmetical multiplication (common fractions)	9
Arithmetical division (common fractions)	2
Fractional equations	6
Chemical equations	134
Percentage	29
Denominate Numbers	173
Substitution in Formula	9
Ratio and Proportion	77
Algebraic addition	2
Algebraic subtraction	0
Algebraic multiplication	0
Algebraic division	0
Highest Common Divisor	1
Clearing equations of fractions	6

TABLE IX.

Denominate Number Units Used in Problems.

calories	12	cubic feet	5
c. c.	10	cubic meter	2
cm	2	degrees	17

gallon	1	metric ton	1
grams	53	milligrams	5
hour	2	minute	1
kilograms	9	pound	5
liters	32	square centimeter	2
mm	9	ton	3
meters	2		

There were fifty-nine cases of the type just indicated. There were eighteen cases involving simple ratios. It is very evident that the principles upon which ratio and proportion rest should be thoroughly understood and be thoroughly usable by the pupil.

Fractional equations were used very little. All of the six cases found might be considered as cases of compound proportion. However, the solution in each case involved substitution in a formula and use of the equality sign, so it was deemed best to list them as above. It is interesting to note, however, that in all of the cases, the denominators were whole numbers of not more than three unit places.

There were twenty-nine cases of percentage found among the problems. Without exception these were simple applications of the principles of percentage. Nine were cases of base and rate given to find percentage; eight of percentage and rate to find base; and twelve of percentage and base to find rate.

As was to be expected, there was found a wide use of denominate numbers. Fully 90 per cent of the problems involved denominate numbers in some form or other. Nineteen different units were used—with frequencies as indicated in Table IX. Usually, the problem was stated in units of some kind and their use was limited to manipulation merely and did not involve changes from higher to lower terms or from one system to another. This was true in sixty-three cases; in nine cases, a change from a higher to a lower denominator was required; in four, from a lower to a higher; in three, a determination of volume was required; in two, a change from the English system to the Metric and in two more, from the Metric system. Frequently, where changes were required from English to Metric or the reverse, the equivalent values were given directly.

There is very little use made of addition, subtraction, multiplication and division in algebra. These processes are used quite extensively, however, in an arithmetical way. This is particularly true of multiplication. Its frequency, both for its use with integers alone and with decimals, is large. Division and addition are used much less frequently.

The range of additions is indicated by the following facts: In sixty cases, additions were made of integers of 7 places or less to another integer of 7 places or less. In 30 cases, three integers of 6 places or less were added while in one case four numbers with 5 places of digits were added. Apart from these, additions of integers and decimals were found. In all, forty-two cases appear. Decimals of more than four places do not appear. The most complicated addition is that of three numbers consisting of one place digit and three decimals (as, 8.456), two place digits and three decimals, and three decimals. In only four cases was a decimal added to a decimal; in the remaining cases a decimal is added to a whole number.

The range of subtractions is very narrow. In the twenty-one cases reported, the most complicated is the subtraction of a six place integer from another of the same kind. The most common (eleven cases) consisted of the subtraction of a one place from a two place integer.

In the 235 cases of multiplication, 93 consisted of the multiplication of two place integers by a one place. In 156 cases, the operation involved nothing more complicated than a four place integer by a six place (of which there was one case). Three integers were multiplied only three times and no cases occurred of the multiplication of more than three. In the matter of decimals sixty-three cases were found in which a decimal or mixed decimal is multiplied by an integer. Of these, the decimal is never more extended than four places and the mixed decimal than five places in the integral portion and three places in the decimal portion. In sixteen cases a mixed decimal is multiplied by a mixed decimal, the most difficult being the multiplication of a number composed of two places in the integral portion and three places in the decimal by another composed of three places in the integral portion and two places in the decimal.

In the 139 cases of division, forty-six consisted of the division of one integer by another. These varied from one to twelve place integers. There was but one twelve place integer, one nine place integer and two eight place integers. Most of them were one, two or three place integers. There were twenty-five cases where a decimal was divided by an integer. In all but one case, the decimal was mixed. The most complicated of these were composed of eight places in the integral portion and one place in the decimal (1 case), 2 places in the integral portion and 4 places in the decimal (1 case), and 2 places in the integral portion

and 3 places in the decimal (1 case). There were twenty-six cases where an integer was divided by a decimal. In all but four of these the decimal was mixed, the most complex being composed of 3 unit places and 2 decimal places as 456.27. The numerator in most of these cases was not larger than a 5 place integer; in 2 cases, a 6 place integer. There were forty-two cases where a decimal was divided by a decimal, and in twenty-eight of these, both numerator and denominator were mixed decimals. A typical case is here given—34.598 divided by 27.4. The most complicated case is that of a number composed of 6 unit places and 2 decimal places divided by a number composed of 3 unit places and 1 decimal place.

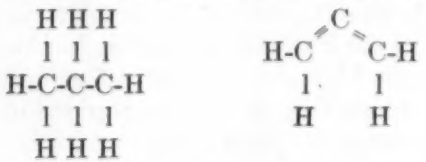
Three important things remain for consideration—chemical symbols, chemical formulas and chemical equations. All have naturally a very extensive use and consequent emphasis upon the mathematics involved. With these is bound up the question of valence as well as that of atomic and molecular weight. A symbol stands for the name of an element, for one atom of that element and for one gram-atom of that element. More than one atom is indicated by a number placed at the right and a little below the symbol. A formula stands for the name of a compound, for one molecule of that compound, for one gram-molecule or for one gram-molecular volume, if a gas. Two or more molecules are represented by a number placed before the formula. An equation gives the substances which react with each other and the products formed, the number of grams of each substance or if the substances are gases, the number of gram-molecular volumes.

The mathematics used here is relatively very simple. Representing more than one atom or more than one molecule involves the simple principle of algebraic coefficients, the location of the figure in the case of the symbol being a mere matter of convenience. With atomic weight, the problem is very similar. Here the numbers are generally not simple but larger and often mixed decimals, as 35.46 for chlorine. The molecular weight is obtained by adding the atomic weights of the elements in the formula (or the multiples) of the atomic weights.

Chemical equations have some things in common with algebraic equations but they differ materially in several respects. We will concern ourselves with the mathematical phases only. The use of the equality sign, transfer of terms from one side of the equation to the other and general handling of the equation,

all conform to mathematical principles. Very few cases were found, however, where transfer of terms, simplification of the equation or general manipulation was used. Balancing a chemical equation is, however, a distinct process. It involves a simple numerical relation, keeping the number of atoms of each kind on one side of the equation the same as the number on the other. The success of this, of course, depends upon knowing the products of the reaction and the use of coefficients in expressing the formulas of products and their quantities. Analysis, therefore, shows that the mathematics used consists of ratios of simple numbers, multiplication of mixed decimals by simple numbers, addition of these products, algebraic coefficients and simple manipulation of equations. Nearly 1,000 different symbols and formulas were used and about 500 equations.

Valence is the property possessed by elements or radicals of combining with or replacing other elements or radicals in definite and constant proportion. The degree of this property, usually indicated by the number of monad elements, represented by hydrogen, with which an atom or radical can combine, is the meaning which concerns us in this study. Thus, valence is a simple, numerical relation and involves nothing beyond simple cases of ratio or proportion. A careful study of the formulas found in the text shows that the following are typical of the ones used almost entirely—Hcl, Na_2SO_4, $Cacl_2$, H_3PO_4, $Alcl_3$, H_4SiO_4, Al_2S_3, $CaSO_4$, $Ca_3(PO_4)_2$. The simple ratios involved here are 1:1, 1 and 1:2, 2:1 and 1, 1 and 1 and 1:3, 3:1 and 1 and 1, 1 and 1 and 1 and 1:4, 3 and 3:2 and 2 and 2, 2:2, 2 and 2 and 2:3 and 3. Only in the organic compounds do we find any noticeable departure from such simple ratios as are here indicated. Even then, whether it be a question of stationary valence or multiple bond, the basis is still simple, as, for example, in the structural formulas for C_2H_4.



Carbon having a valence of 4; has 4 arms or bonds for each atom. These must be satisfied for each atom, by union with bonds of other elements or with bonds of other carbon atoms. Even in the second case above, that of C_2H_4, the double bonds

required to satisfy conditions, do not require anything but simple ratios.

Conclusions.

1. Summarizing, we may say that the study shows a rather limited use of mathematics in a first-year course of chemistry based upon the text examined—limited in the sense of the number of processes used and in the relatively simple nature of most of those processes. Algebra and geometry have a very limited use, though it is possible to use algebra occasionally when arithmetic might not be quite as direct.

2. A good working knowledge of the important principles of the following processes is desirable: addition, subtraction, multiplication, division, fractions, both common and decimal, graphs of simple curves and proportions, percentage, surface measure, volume measure, metric system, simple progression (used but slightly), ratio and proportion, and square root.

3. For solving the problems given, a thorough knowledge of the fundamental processes of arithmetic is necessary. This should hold, no matter whether integers, decimal fractions or common fractions are used. Simple equations of one unknown and simple fractional equations should be mastered. The pupil should have thorough training in percentage, denominate numbers and ratio and proportion.

4. A knowledge of the fundamental processes in algebra seems desirable, though by no means as essential as might be inferred. Aside from their use with algebraic equations, themselves seldom found, they are scarcely ever used.

5. It appears entirely feasible to construct upon the basis of the facts here displayed a simple set of exercises or preferably practice tests for chemistry students who would be required to attain efficiency in them outside of the class. They might even be prepared and distributed to high school students expecting to take Freshman chemistry in institutions using this text. It appears very clear that such a device would give a much better hold upon preparatory mathematics than would an additional year of algebra, trigonometry or solid geometry.

CLASS ROOM SAYINGS.

Where does the food go, after leaving one's stomach? It passes into the testaments.

Rain is formed by the condensation of the humidity in the air.

People should be very careful not to get humane (ptomaine) poisoning.

THE CONVENTIONAL EXAMINATION IN CHEMISTRY AND PHYSICS VERSUS THE NEW TYPES OF TESTS: PART 1

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In the attempt to improve upon the traditional examination in physical science it is important that we keep in mind certain facts with respect to the intelligence of pupils and the present practice of teachers in giving school marks:

I. According to the psychologists*, pupils may be distributed over three different scales; one represents *inherited capacity*, one represents *ability*, and one represents *specific performance*.

1. Inherited capacity has to do with the "start in life." We do not measure *inherited capacity* directly in school work.
2. If the educational conditions are favorable in the home, school, etc., the rank of the individual on the *ability* scale will approximate his true rank on the *inherited capacity* scale.
3. If a student's ability is his *ability-to-do* then what he does represents his *specific performance;* that is, the problem that he solves, the experiment that he performs, etc.
4. These *specific performances* may be measured in school practice. *It is these that school marks ought to measure.*
5. We do not *measure* ability, however; we *estimate* it by the specific performances of the student.
6. There are two important characteristics associated with the performances of a student:
   (a). Specific performance is *unstable* and *one test* does not give an accurate estimate of the ability of a given student.
   (b). The average of several (3 or 4) performances is relatively stable. The tables below show these two facts.

TABLE I: CHEMISTRY

A table showing the results obtained on a series of tests given near the end of the course, 1920-1921:

No. 1

No.	Name and rank on composite of 12 tests. Student.	Rank on Stanford revision of Binet-Simon test.[1]	Rank by (1) quality of experimental work.	Rank by (2) accuracy in solution of chemical problems	Rank by (3) quality of a paper written upon a chemical industry.
1	A	1 (137)	1	1	1
2	B	4 (115)	4	4	4
3	C	2.5 (119)	2	2	2
4	D	2.5 (119)	3	3	3
5	E	5 (113)	5	7	7
6	F	7.5 (104)	8	9	9
7	G	6 (107)	7	8	6
8	H	7.5 (104)	6	6	8
9	I	9 (95)	9	5	5

[1]Tests given January to March, 1920.

No. 2

No.	Name and rank on composite of 12 tests. Student.	Rank by (4) ability to assemble chemical information in graphical form.	Rank on (5) technical information test. (100 items.)	Rank on (6) Stenquist mechanical ability test (Picture test)	Rank on (7) silent reading test (Burgess PS-1)
1	A	1	3	4	1
2	B	2	4	3	3
3	C	7	8	7	8
4	D	5	6	6	4
5	E	8	1	1	6
6	F	4	2	2	5
7	G	6	7	8	7
8	H	3	9	9	2
9	I	9	5	5	9

II. A school mark or an examination grade should have the same meaning to student, teacher, parent, and administrator.

No. 3

No.	Name and rank on composite of 12 tests. Student.	Rank (8) chemistry test M C (Biography)	Rank (9) chemistry test No. 1 (Formulas symbols, etc.)	Rank (10) on chemistry test No. 2 (Range of information)	Rank (11) on chemistry test No. 3. (Problem-solving)	Rank (12) on chemistry test C-1
1	A	1	1	2	1	1
2	B	4	5	3	5	3
3	C	2	2	1	4	2
4	D	3	3	5	2	6
5	E	5	4	4	3	5
6	F	7	8	6	8	8
7	G	6	6	8	6	4
8	H	8	7	7	7	9
9	I	9	9	9	9	7

TABLE II: PHYSICS

A table showing the results obtained on a series of tests given near the end of the course, 1920-1921:

No. 1

No.	Name and rank on composite of 14 tests. Student.	Rank on Stanford revision of Binet-Simon test.[1]	Rank (1) on silent reading-test (Burgess PS-1)	Rank (2) on Stenquist mechanical-ability test	Rank (3) on problem solving test V	Rank (4) on physics test MC (Biography)
1	A	6 (109)	4	3	1	1
2	B	9 (93)	9	1	4	2.5
3	C	3 (114)	2	4	2	6.5
4	D	4 (111)	6	2	3	8
5	E	7 (108)	3	8	5.5	5
6	F	1 (129)	7	6	8	2.5
7	G	2 (117)	1	5	5.5	4
8	H	5 (110)	8	7	9	6.5
9	I	8 (98)	5	9	7	9

[1]Tests given January to March, 1920.

No. 2

No.	Name and rank on composite of 14 tests Student	Rank (5) on physics test X (Completion test on laws)	Rank (6) on physics test A (True-false on mechanics heat)	Rank (7) on physics test L P (Completion, whole course)	Rank (8) on physics test S (Problem solving)	Rank (9) on physics test T F (True-false, whole course)
1	A	1	1	1	1	1
2	B	2	4	2	7	3
3	C	3	3	4.5	2	4
4	D	7	6	3	4	2
5	E	4	2	6	8	8
6	F	5.5	5	7	5	6
7	G	5.5	7	4.5	3	5
8	H	9	9	8	9	7
9	I	8	8	9	6	9

No. 3

No.	Name and rank on composite of 14 tests. Student.	Rank (10) on technical information test (100 items)	Rank (11) by quality of experimental work.	Rank (12) by quality of home written work.	Rank (13) by quality of paper on special topic.	Rank (14) on accuracy of numerical problems
1	A	2	1	1	1	1
2	B	1	2	2	2	3
3	C	3	5	6	3	4
4	D	5	9	8	4	6
5	E	7	3	3	9	2
6	F	4	4	4	6	5
7	G	6	8	9	7	8
8	H	8	6	5	8	7
9	I	9	7	7	5	9

There are some fundamental objections to the marks as ordinarily given upon the traditional examination papers:

1. There are large variations in teachers' marks of the same student, in the same subject, on the same examination. The variation is practically never less than 5% and is often as much as 15%. Few teachers can use a marking interval smaller than 8%.
2. The relative difficulty of the questions is unknown. One investigation shows a range of variation of 10 to 298 points on a single examination.
3. Due to "passing" grades and other causes, the usual examination grades are not consistent and are not distributed in accord with the known range of specific performance of students.
4. The mark given on a conventional examination does not measure *any single thing*. It measures a variety of specific performances involving four factors; i. e., quality of product, difficulty of task, time consumed, and quantity of product or work done.
5. In physical measurement the thing to be measured must be consistent, so that one unit is like another. Such consistency will be secured in educational measurements by arranging the test to measure a *single variable*. The other factors should be kept relatively constant.
6. Some additional objections to the traditional examination are:
   (a). The strain produced upon students and teachers.
   (b). The delay of promotion until the examination period.
   (c). Only a few points of subject matter are covered in the test.
   (d). It requires little care in preparation on the part of a teacher.
   (e). It requires *too much* time to score.
   (f). Students spend too much time in writing and too little in thinking.
   (g). It has *no legitimate use* as a basis for determining how many pupils should fail in a subject.

III. Some advantages for the new types of test are:

1. The definite testing of some selected performance without leaving much to the personal equation of the examiner.
2. Freedom from strain of the two or three-hour examination.

3. Absence of the great labor of grading conventional "essay" examination papers.
4. Compels a decision as to what is being taught before undertaking to measure it.
5. The order of difficulty approximates the varying abilities of the class.
6. The wide range covered makes it possible to discover what topics have not been effectively taught.
7. The labor of scoring is reduced to a minimum.
8. Permits pupil to spend time thinking rather than in the mechanical work of writing.
9. Eliminates handwriting, style, etc., as disturbing factors in testing a specific performance.
10. Permits pupils to be tested on a specific performance, and excludes consideration of general ability in English composition.
11. Makes possible construction of another test next year of entirely different questions but of the same difficulty for testing a given performance.
12. The pupils *enjoy* these tests and the personal judgment of the teacher is not a factor in deciding the grade.

The chief disadvantages of the new tests are:

1. The units of thought may be too short.
2. The questions must be selected with great care, otherwise the poor student will make a high score.
3. Each pupil must have a copy of the test. This raises the item of expense, which is a factor in most schools.
4. Most teachers do not understand the methods to be used in preparing these tests, and many of them do not care to learn.

* References:

1. Burgess, May Ayres. The Measurement of Silent Reading (1921), Russell Sage Foundation, New York.
2. Rugg, H. O. Teachers' Marks and the Reconstruction of the Marking System. *Elementary School Journal, 18*, 701-719 (1918).
3. Trabue, M. R. Greater Objectivity in School Examinations. Bulletin of New York Soc. for Exp. Study of Ed. *2*, No. 18, (1920).

PLATINUM PRODUCTION IN 1920.

Estimates of the production of crude placer platinum in the United States in 1920 give Alaska, 27 ounces; California, 656 ounces; Oregon, 23 ounces; and Washington, 8 ounces.

Refiners reported a production of 41,544 troy ounces of new platinum metals in 1920, of which 36,015 ounces was platinum, 418 ounces iridium, 409 ounces osmiridium, 4,309 ounces palladium, and 393 ounces of the minor metals, including rhodium and osmium. This represents a decrease of 3,565 ounces, as compared with the production in 1919.

There were also produced 57,710 ounces of secondary platinum metals in 1920, of which 51,255 ounces was platinum, 3,355 ounces iridium, and 3,100 ounces palladium. This represents a decrease of 3,806 ounces, as compared with the production in 1919.

The platinum metals imported for consumption in 1920 included 80,955 ounces of crude platinum and unmanufactured ingots, bars, sheets, etc., of which approximately 58,009 ounces is believed to be refined metal; 4,718 ounces of iridium; 4,473 ounces of osmiridium; 6,944 ounces of palladium; 2,675 ounces of the minor metals; and 781 ounces of manufactured platinum ware. The imports of platinum for consumption in 1920 were 32,492 ounces greater than in 1919.

The consumption of platinum metals in the United States in 1920 was 141,041 ounces, of which the jewelers took 57 per cent, the electrical industry 19 per cent, the dental industry 11 per cent, the chemical industry 10 per cent, and miscellaneous uses 3 per cent.

THE IMPROVEMENT OF CHEMISTRY AND PHYSICS INSTRUCTION IN AMERICAN HIGH SCHOOLS.[1]

By Earl R. Glenn,

The Lincoln School of Teachers College, Columbia University.

I

In discussing the improvements that are possible in chemistry and physics instruction, it is well to keep in mind the fact that the higher institutions still exercise a large measure of control over the course taken by a high school student.

According to a government bulletin[2] the distribution of prescribed units for entrance to state colleges and universities in 1915 was as follows:

(a) Thirty-six states require three years or more of English
(b) Thirty-seven states require two years or more of mathematics
(c) Thirty states require two years or more of foreign language
(d) Two states require as much as two years of science and none require more
(e) Nineteen states require one year of science. In more than one-half of the states a student may enter the state colleges or universities with no previous science training.

So long as these requirements exist to the disadvantage of science instruction, a large percentage of the high school population will never be enrolled in science classes, no matter how interesting or valuable the science courses may be.

II

As chemistry and physics teachers, we should adopt such a science sequence as that suggested by the N. E. A. Commission on the Reorganization of Secondary Education.[3] Until the science teachers *do agree upon some science sequence* it will not be possible to compete with English, foreign language, and mathematics, or to obtain information about the learning processes or the "grade-placement" of the extensive and complicated subject matter of science.

III

At the present time, the teaching of chemistry and physics has reached such a state of development that we may well discard some of the methods of teaching used in higher institutions that have been found through extensive trial by competent observers not to function in high school instruction. To this end the high school science lecture room should be omitted from the designs of all new high school buildings and there should be substituted for it a unit science laboratory which is designed

[1] Given at the University of Pennsylvania, April 7, 1921, during Schoolmen's Week.
[2] Capen, S. P. Accredited Secondary Schools in the U. S. Bull. 1916, No. 20, U. S. Bur. of Ed.
[3] Caldwell, Otis W., and Committee, "Reorganization of Science in Secondary Schools," Bulletin 1920, No. 26, Government Printing Office, Washington, D. C., Price, 10 cents.

for class demonstration or individual laboratory work, all of which in a single room is under the control of the class teacher. Such designs will be found in the Central Technical School, Toronto, Canada, The Lincoln School of Teachers' College, New York, and Girard College, Philadelphia.

IV

As teachers we are too uncritical of the subject matter that we use for instruction. From time to time both laboratory manuals and textbooks need to be evaluated in an objective way. The content of such influential examinations as those given by the College Entrance Examination Board should also be examined critically. This has recently been done for chemistry[4] with the following conclusions:

DISTRIBUTION OF THE SUBJECT MATTER OF CHEMISTRY QUESTIONS, 1911–1920.

Topic	Per cent of Frequency		
	Old Plan	Comprehensive	All
Common non-metallic elements	8.0	9.0	8.4
Common metallic elements	4.2	2.6	3.6
Common acids	7.8	10.2	8.7
Salts, oxides and hydroxides	17.9	7.6	13.8
Gases (not elements)	7.5	6.1	6.9
Organic substances	2.5	2.6	2.5
Identification (laboratory)	11.6	7.6	10.0
Quantitative experiments	0.55	0.6	0.6
Equations	12.4	17.8	14.5
Reversible reactions	1.0	2.2	1.4
Laws and Theories	6.3	5.2	5.9
Electrolytic Dissociation Theory	3.1	5.2	3.9
Chemical arithmetic:			
Percentage composition	1.5	2.3	1.8
Weights of substances concerned	1.7	1.7	1.7
Volumes of gas from a reaction	3.0	3.2	3.1
Energy changes	0.55	0.9	0.7
Oxidation and reduction	1.7	1.5	1.7
Solutions	1.5	2.9	2.1
Others	7.2	10.8	8.7
Total	100.0	100.0	100.0

We have made a similar study for physics and while the complete summary is not yet published, the following facts appear to be significant for "the old plan" questions:

a. Questions and parts of questions for ten years distributed by definitions of terms, laws, physical phenomena to be explained, problems, and experiments.

b. The equations for Ohm's law appear with a frequency of 19 times that of the equation for relative humidity.

c. Problems involving the gas laws appear seven times as often as those involving the variation of light intensity with distance.

[4] Gerry, Henry L., "College Entrance Examination Board Questions in Chemistry," SCHOOL SCIENCE AND MATHEMATICS, 20 (1920), 845-50.

d. Twenty different formulas were asked for only once in a period of ten years.

Division	Mechanics	Heat	Electricity	Sound	Light	Total
Physical terms	19	9	0	5	3	36
Physical phenomena	11	5	16	11	35	78
Laws and facts	35	5	5	14	7	66
Problems	43	19	32	8	25	127
Experiments	7	3	2	4	1	17
Total	115	41	55	42	71	324

V

Since our school promotions depend upon tests of one sort or another it is time that we, as teachers of exact science, should make our examinations less burdensome to teacher and pupil and somewhat more exact.

This is important for three reasons:

a. Teachers marks are unreliable.

b. The marks given by teachers show a striking variability.

c. Teachers are inconsistent in the distribution of class marks.

The investigations which support these statements are summarized by H. O. Rugg in the Elementary School Journal[5] for May, 1918. Any who wish to examine the evidence in detail should read this valuable article. In all probability these three fundamental criticisms of our marking system apply to a considerable extent to teachers of chemistry and physics. Some new tests are now being devised for these subjects and any who are interested should communicate with the author. The methods used in devising and scoring tests are discussed in the references on educational measurements which are listed below.

VI

Some Books on Educational Tests.

1. Burgess, May Ayres, "The Measurement of Silent Reading," 1921, Russell Sage Foundation, New York.
2. Caldwell, Otis W., "Science in the Gary Schools," 1919, General Education Board, New York.
3. Monroe, Walter S., "Measuring the Results of Teaching," 1918, Houghton.
4. Rugg, Harold O., "Statistical Methods Applied to Education," 1917, Houghton.
5. Terman, L. M., "The Intelligence of School Children," 1919, Houghton.
6. Wilson, G. M. and Hoke, K. J., "How to Measure," 1920, Macmillan.
7. Yockum, C. S. and Yerkes, R. M., "Army Mental Tests," 1920, Holt.

[5] Rugg, H. O., "Teachers' Marks and the Reconstruction of the Marking System," Elementary School Journal, 18 (1918), 701-19.

DO HIGH SCHOOL PUPILS DISLIKE MATHEMATICS?

By W. G. Gingery,

Shortridge High School, Indianapolis, Ind.

Professional educational agitators, who retain their seats in chairs of pedagogy by making startling assertions about the subjects of the classical curriculum, frequently assert that children do not like mathematics and therefore ought not to be required to study it. Having had opportunity to ask a large homogeneous group what their preferences were among the subjects they had taken in high school, we are submitting the results of their answers.

The questionnaire was passed out through the session rooms and no mention was made of the department that gave the questionnaire. The pupils had been in high school one semester and all were required to take English and mathematics. One other full time subject and music and gymnasium were required, and in some exceptional cases a fourth full time subject was allowed.

The pupils were asked to write their subjects in the order of their preference for them. Not all pupils recorded their part time subjects so the record for art, gymnasium, and music is not significant.

The following table contains the results of the questionnaire. The last column is a duplication of some of the third, fourth and fifth choices, that is, if a pupil had three subjects his third choice would also be his last choice.

Subject	Total	First Choice	Second	Third	Fourth	Fifth	Last
English	301	104	92	80	25		51
Mathematics	302	100	101	76	23	2	52
Latin	194	39	49	70	32	4	51
French	26	8	9	8	1		2
Spanish	10	4	2	4			
History	69	25	15	17	12		17
Art	21	9	7	2	3		4
Gymnasium	82	5	10	16	49	2	47
Physiography	19	6	1	10	2		3
Music	121	9	20	21	71		79

In order to get a comparative score a first choice is counted one, a second, two, and a third, three, and so on, and the result averaged. No account is taken of the column of last choices. The scores follow:

English	2.086	History	2.232
Mathematics	2.099	Art	1.952
Latin	2.551	Gymnasium	3.402
French	2.077	Physiography	2.421
Spanish	2.000	Music	3.273

These pupils were further classified according to the ten teachers who taught them mathematics. The following table indicates the result of this classification:

Teacher	Total	First Choice	Second	Third	Fourth	Fifth	Last
A	24	8	9	6	1		2
B	21	6	12	3			2
C	20	5	6	6	2	1	5
D	33	12	14	5	2		5
E	50	10	20	15	5		9
F	43	15	15	11	3		10
G	45	21	10	10	4		5
H	20	11	5	3	1		3
I	19	4	4	8	2	1	6
J	10	4	2	4			1

The scores of the teachers on the preceding basis follow:

A	2.000	D	2.212	G	1.933
B	1.857	E	2.300	H	2.200
C	2.400	F	2.093	I	2.570
		J	2.000		

The pupils were further asked to tell why they liked best the subject placed first. The answers to this question were not especially illuminating. Of the one hundred and one who preferred mathematics, twenty indicated that it was because they had good teachers. Some said it was because the subject was so definite, and some because it was like a puzzle, and in some cases these two answers came from the same class. Some said it was hard but they could master it and some thought it easy. These answers were of interest to the teachers in charge and in some cases were of profit to them. They would not, however, be of much general interest.

The conclusion seems to be, without question, that mathematics is neither more nor less disliked than other required subjects of the first year high school course.

TOPOGRAPHIC AND SUPERPOWER SURVEYS.

A program for completing the topographic map of the United States for the use of this generation was outlined in last year's report of the Director of the United States Geological Survey, but Congress has not yet approved that plan, though it promises to get results most expeditiously and economically. The power program, also presented in the report for last year, has found more encouraging support, as the superpower survey of the eastern industrial zone has been authorized by Congress and is now under way. It is hoped that the engineering report on this survey will commend itself so highly to Congress that similar investigations of other parts of the United States may be authorized before the demand for unified electrification becomes so urgent as it has already become along the North American seaboard.—Forty-first Annual Report, Director United States Geological Survey.

PROBLEM DEPARTMENT.

Conducted by J. A. Nyberg,
Hyde Park High School, Chicago.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and solve problems here proposed. Problems and solutions will be credited to their authors. Each solution, or proposed problem, sent to the Editor should have the author's name introducing the problem or solution *as on the following pages.*

The Editor of the department desires to serve its readers by making it interesting and helpful to them. If you have any suggestion to make, mail it to him. Address all communications to J. A. Nyberg, 1039 E. Marquette Road, Chicago.

LATE SOLUTIONS.

691. *J. Pickett, Ranger, Texas.*
692. *DeWitt T. Weaver, Sistersville, W. Va.*
694. *E. Tabor, Upper Lake, California.*

SOLUTION OF PROBLEMS.

690. *Proposed by Moe Buchman, Brooklyn Boys' High School Mathematics Club.*

A train going at a certain rate meets with an accident which delays it a hours. After the accident the train's rate is one nth less than its original rate, and it reaches its destination b hours late. If the accident had occurred a distance c miles farther away, the train would have been d hours late. What was the train's original rate?

This problem was solved last May, and a question was raised about an incorrect solution. The error lies in the fact that while *distance* equals the product of *rate* and *time*, it is not true that a difference of two distances equals the product of the difference in the rates by the difference of the times; that is, $d_1 = r_1t_1$ and $d_2 = r_2t_2$ but $d_1 - d_2 \neq (r_1 - r_2)(t_1 - t_2)$. The following very brief solution is suggested by *Norman Anning.*

To go c miles takes $b-d$ hours longer when the rate is one nth less. Hence, if x is the rate, then $c/x + (b-d) = c/x(1-1/n)$ and $x = c/(b-d)(n-1)$.

696. *Proposed by Walter R. Warne, Pennsylvania State College, State College, Pa.*

Construct the triangle given $\angle A$, a and the sum of b and c.

Solution by A. H. Lord, Classical High School, Lynn, Mass.

Let $MB = b+c$. At M construct an angle equal to $\frac{1}{2}\angle A$. With B as a center and a as radius draw an arc cutting the other side of the angle at C and C'. From C and C' draw lines CA and C'A' to MB making $\angle MCA$ and $\angle MC'A'$ equal to $\frac{1}{2}\angle A$. Triangles ABC and A'BC' are the required triangles. According as BC is less than, equal to, or greater than the altitude from B to MC there will be no, one or two solutions.

Similarly solved by *N. Barotz, New York City; Moe Buchman, Brooklyn, N. Y.; H. E. A. Lazott, Nashua High School, N. H.; Morris Monsky, Brooklyn, N. Y.; Irene Ricard, Notre Dame Ladies' College, Montreal; G. Rousseau, Ecole Polytechnique, Montreal; Forrest Young, Redlands, Cal.; and the Proposer.* Another solution consists in drawing upon the line a as a cord a segment of a circle subtending an angle equal to $\frac{1}{2}\angle A$. This was done by *T. E. N. Eaton, Redlands, Cal.;* and by *Doris Brophy, Notre Dame Ladies College, Montreal. Carl E. Howe, Blue Ridge College, New Windsor Maryland,* and *W. R. Warne* showed by trigonometric equations how b and c could be expressed in terms of CosA and $(b+c)$. *Abe Poll,* student at the *Milwaukee State Normal, J. Pickett, Ranger, Texas; E. Tabor, Upper Lake, Cal.,* based their solutions on the idea that the vertex A must lie on an ellipse of which BC, or a, is the line joining the two foci. The Editor wishes to add that while the given parts may be so chosen that no solution is possible, still it is hardly correct to say that two solutions are ever possible for the two triangles ABC and A'BC' in the solution above are congruent since AC = A'B and A'C' = AB and BC = BC'. (This is a very good exercise for Book I in geometry). Trigonometrically

the problem is solved best by using the relations $\cos\frac{1}{2}(B-C) = (\sin\frac{1}{2}A).(b+c)/a$, $b-c = a\sin\frac{1}{2}(B-C)/\cos\frac{1}{2}A$.

697. *Proposed by Earl W. Martin, Wilmington, Ohio.*

Squares with sides 1, 3, 5, 7, . . . are placed on a line so that their bases lie corner to corner. Show that their upper left vertices lie on a parabola and are at integral distances from the upper right hand vertex of the first square; and thus derive formulas for use in obtaining pairs of sides of right triangles whose sides are integral.

Solution by J. Pickett, Ranger, Texas.

Let A be the right upper vertex of the first square, and B the left upper vertex of any square in the series, whose side is $2n-1$. Craw a line MN perpendicular to the base line and one unit to the left of the first square. Produce the upper base of the first square to cut MN at D, and to cut the left side of the last square at C. Draw BE $\perp$MN, cutting MN at E. Draw AB.

Then $BC = 2n-2$, $AC = n^2-2n$, and $BE = n^2-2n+2$. $AB^2 = BC^2+AC^2$ or $AB = n^2-2n+2$. Since AB = BE, B lies on a parabola whose focus is A and whose directrix is MN. The numbers $2n-2$, n^2-2n, and n^2-2n+2 are integral for all integral values of n and are the two sides and hypotenuse of a right triangle.

Similarly solved by Moe Buchman; Orville A. George, Mason City Junior College, Iowa; Morris Monsky; D. T. Weaver. By using other lines for reference the sides of the triangle would be $2n$, n^2-1, n^2+1, or $2n+2$, n^2+2n, n^2+2n+2. These were obtained by *N. Barotz; T. E.N. Eaton; Michael Goldberg, Philadelphia, Pa.; Carl E. Howe; H. E. A. Lazott; Gabriel Rousseau; E. Tabor.* In School Science and Mathematics, June, 1920, page 562, is a further study of the right triangle with integral sides by *R. R. Knowles, Industrial Arts H. S., Sterling, Colo.*

698. *Proposed by E. E. Waldrep, Birmingham Southern College, Ala.*

Find the ages of the people in the statement: When I was born my sister was one-fourth mother's age but she is now one-third father's age. I am now one-fourth mother's age but in four years I shall be one-fourth father's age.

Solution by Abe Poll, student, Milwaukee State Normal.

Suppose that I am now x years old; then my mother is $4x$ years old; and in 4 years my father will be $4(x+4)$ so that he is now $4(x+4)-4$ in age.

When I was born my mother was $4x-x$ or $3x$, my sister was $3x/4$, and she is now $x+3x/4$ years.

My father is now $3(x+3x/4)$ and he is also $4(x+4)-4$. Hence $4x+12 = 21x/4$; $5x = 48$. The ages now are: mother 38.4, father 50.4, sister 16.8, I 9.6.

H. E. A. Lazott offers the following solution by the geometry class in the *Nashua High School, N. H.*, as an exercise in analysis.

	When I was born	Now	In four years
I	0	x	$x+4$
Sister	y	$x+y$	$x+y+4$
Mother	$4y$	$4x$	
Father		$3(x+y)$	$4x+16$

The equations then are $4x-x = 4y$ and $4x+16-4 = 3x+3y$, giving $x = 9.6$ and $y = 7.2$.

Also solved by *Katherine S. Arnold, Milwaukee-Downer College; N. Barotz; Wm. H. Brown, Principal, Amherst (Mass.) High School; Moe Buchman; T. E. N. Eaton; Ruth Foley, Danville H. S., Ill.; O. A. George; I. Ginsberg, Newark, N. J.; M. Goldberg; Carl E. Howe; Lois C. Ketcham, Chester, Va.; Harriet Kibler, New Castle, Ind.; Daniel Kreth, Wellman, Iowa; A. H. Lord; M. Monsky; Willis Patton, Mason City, H. S., Iowa; Fern Poulson, sophomore, Twin Falls H. S., Idaho; Irene Ricard; J. L. Riley, Texas Agricultural College, Stephenville, Tex.; Gabriel Rousseau; Hazel E. Schoonmaker, Augustana College, Rock Island, Ill.; E. Tabor; Walter R. Warne; D. T. Weaver; Ethel M. Weymouth, Wells H. S., Maine; Henry L. Wood, Corinth, N. Y.;* and by the following students of the *Redlands High School, Cal.: May Buckwell, '23; Eleanor Burns; Ralph*

Cope, '24; *John Daniels*, '23; *Harry Deming*, '22; *Maurice Elliott*, '21; *Winford Lane; Junior Meserve; Christine Mitchell; Mary Murray*, '22; *Engle Randolph*, '24; *George Shinn*, '22; *Virginia Shinn*, '24; The Editor wishes he had room here for a very interesting letter from *Thomas E. N. Eaton*, instructor at the *Redlands High School*, telling how he uses the problem department in his classes. Among other things, in any regular examination a pupil is excused from one of the exercises if he has solved correctly and without help any problem proposed in this department.

699. *Proposed by the Editor.*

In a given $\triangle ABC$ find a point P such that the three lines PA, PB and PC shall make given angles with each other.

Solution by E. Tabor, Upper Lake, California.

Upon AB as a chord construct a circle so that the angle inscribed in the segment (which falls on the triangle) will equal the given angle APB. Similarly on BC as a chord construct a segment containing the given angle BPC. The intersection point of the two circles is the desired point. The problem will always be possible when the sum of the given angles is a perigon, and when each of the angles at P is greater than the corresponding angle of the triangle (opposite the same side).

Also solved by N. Barotz; D. Brophy; M. Buchman; F. A. Cadwell, St. Paul, Minn., with a discussion of the number of possible solutions; *A. H. Lord; J. Pickett; A. Poll; Gabriel Rousseau; H. L. Wood; W. R. Warne.*

Discussion by H. E. A. Lazott, Nashua High School, N. H.

Since the sum of the given angles must be 360°, any two may be drawn opposite any pair of sides of the given triangle and the third angle will be determined by the exterior sides of these two. Hence, there will be in general 6 positions for P from which lines to A, B, and C will include the required angles. If the smallest of the three given angles is greater than the greatest of the three angles of the triangle, then the six points could be found within the triangle. This would happen if the triangle were acute and the given angles obtuse. If two of the given angles be right angles, the pedal points of perpendiculars from the vertices to the opposite sides will be the required points; and in an acute triangle there would be three of them. If only one of the given angles is acute, and one angle of the triangle is obtuse and also smaller than the other two given angles, there will be four solutions; but if the obtuse angle is greater than one of the other given angles, only two solutions are possible.

This problem has suggested another to Norman Anning, which is stated below.

700. *For high school students. Proposed by Smith D. Turner, student, Parkersburg High School, West Virginia.*

In a $\triangle ABC$ the altitude BD is drawn. $\angle ABC = 45°$, $AD = 9$, $DC = 6$; find AB, CB, DB.

I. *Solution by Morris Monsky, Brooklyn Boys' High School, New York.*

Let O be the center of the circumscribed circle; at C draw the tangent CG. Draw $OF \perp AC$, $OE \perp BD$; draw OB and OC.

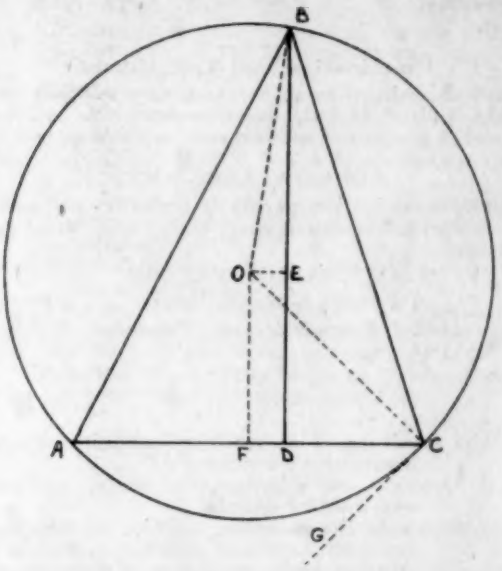
Then $\angle ACG = \angle B = 45°$, and $\triangle COF$ is a right isosceles triangle. $CF = AC/2 = 7.5$; $ED = OF = CF = 7.5$; $OE = CF - CD = 1.5$; $CO = 7.5\sqrt{2}$; $BE^2 = OB^2 - OE^2 = 441/4$; $BE = 21/2$; $BD = BE + ED = 18$. $AB = 9\sqrt{5}$. $BC = 6\sqrt{10}$.

II. *Solution by Martin B. Dickinson, North East H. S., Kansas City, Mo.*

Let $x = \angle ABD$, $y = \angle CBD$; then $9/\tan x = 6/\tan y$ and $\tan(x+y) = \tan 45° = 1$. These two equations reduce to $2\tan^2 x + 5\tan x = 3$. The solution $\tan x = -3$ is impossible, leaving $\tan x = 1/2$. Hence $BD = 18$. By the Pythagorean theorem, $AB = 9\sqrt{5}$ and $BC = 6\sqrt{10}$.

Also solved by *Moe Buchman; Orville A. George; Horace Levoy, Los Angeles H. S., Cal.; Irene Ricard; Gabriel Rousseau; Alexander S. Wiener, Brooklyn Boys' High School;* and the *Proposer.* It is interesting to note that all the solutions were different. *Moe Buchman* drew a line through D parallel to BC, and a perpendicular from D to AB, and then considered the various parts. *O. A. George* and *I. Ricard* used formulas for the area of the triangle to get equations. H. Levoy used the equation $9\tan(135° - x) = 6\tan x$ where $x = \angle C$, but could have improved his

work if he had not looked up the value of x in a table; the numerical value of the angle is unnecessary as only the value of $\tan x$ is needed. *A. S. Wiener* was led to simultaneous quadratic equations but gave a very careful study of the extraneous solutions. *G. Rousseau* circumscribed a circle, extended BD to meet the circle at H, and drew CH.



It is now a year since the custom was adopted of reserving the fifth problem each month for the high school students, and the Editor feels that the results have been both instructive and entertaining. From letters received he knows that more teachers are using these problems than the lists might indicate. The custom will be continued.

PROBLEMS FOR SOLUTION.

711. *Proposed by Norman Anning, Ann Arbor, Mich.* (See problem 699).

Given $\triangle ABC$ and a point P inside. Let $x = \angle BPC$, $y = \angle CPA$, $z = \angle APB$. Prove that if N is the number of ways of choosing the angles A, B, C, then the number of ways of choosing x, y, z is

$$N[1-(A^2+B^2+C^2)/\pi^2].$$

712. *Proposed by Daniel Kreth, Wellman, Iowa.*

Construct the triangle, given the base, a, the difference of the two adjacent angles, and the median to the given base.

713. *Proposed by Morris Monsky, Brooklyn Boys' High School, New York.*

A harmonic and an arithmetic progression have the same pth, qth, and rth terms, a, b, and c respectively. Prove

$$a(b-c)\log a+b(c-a)\log b+c(a-b)\log c = 0.$$

714. *Proposed by C. E. Githens, Wheeling, West Virginia.*

A statue thirty feet in height stands on a base twenty-nine feet high, both vertical to a plane. How many feet from the foot of the base is subtended the greatest vertical angle by the statue, the height of the eyes being five feet vertically above the level plane? Also, what is the tangent of the angle thus subtended? (Not to be solved by calculus.)

715. *For high school students. Proposed by Norman Anning.*

Solve the equations: $x+y^2 = 3$, $x^2+y = 3$.

SCIENCE QUESTIONS.

Conducted by Franklin T. Jones.

The Warner & Swasey Company, Cleveland, Ohio.

Readers are invited to propose questions for solution—scientific or pedagogical—and to answer questions proposed by others or by themselves. Kindly address all communications to Franklin T. Jones, 10109 Wilbur Ave., Cleveland, Ohio.

Examination and Test Papers.

Please send examination papers on any subject or from any source to the Editor of this department. *He will reciprocate by sending you such collections of questions as may interest you and be at his disposal.*

ACKNOWLEDGMENT.

The receipt of examination papers is gratefully acknowledged from Miss Annie C. Cloyd, Oklahoma City, Okla., and Mr. John Lundberg, Goteburg, Sweden.

EXAMINATION PAPERS.

PHYSICAL SCIENCE, GRADE XI.

Departmental Examinations, Province of Alberta.

380.

Time—Two and one-half hours.

Values.		
	1.	"The transmission of vibratory motion through a medium constitutes wave motion."
4		(*a*) Describe the vibratory motion of air particles when transmitting sound.
4		(*b*) Compare the vibratory motion of air particles when transmitting sound with the motion of the particles of water under the influence of a series of waves.
4		(*c*) State in what respects sound waves (1) resemble, (2) differ from light waves.
4	**2.**	(*a*) Define the terms: pitch, quality, resonance, node.
4		(*b*) Compare the vibration frequencies of the fundamental note of a closed organ pipe and its first and second overtones.
10		(*c*) Describe an experiment showing how an analysis of complex sound waves may be made.
2	**3.**	(*a*) Distinguish between illuminating power and intensity of illumination.
6		(*b*) Given a lamp of known candle power, how may the candle power of a second lamp be determined experimentally?
		(*c*) A screen is placed 3 feet beyond a convex lens the diameter of which is 3 inches and focal length 6 inches.
2		(1) If the sun's rays are thrown by a mirror on the lens, what will be the diameter of the area illuminated on the screen?
2		(2) Compare the intensity of illumination of this screen with that of one receiving the sun's rays directly.
	4.	Describe experiments to show that:
4		(*a*) Light travels in a straight line in a homogeneous medium.
4		(*b*) The direction of a beam of light changes as it passes obliquely from a medium of one density to another of different density.
6		(*c*) The refractive indices of lights of different colors differ from one another.
	5.	(*a*) A candle is moved along the principal axis of a concave mirror. In what positions will the images formed be:
2		(1) Real and larger than the object?

2 (2) Real and smaller than the object?
2 (3) Virtual?

(*b*) An object 2 inches in length is placed (1) 6 inches, (2)
4 3 inches from a convex lens the focal length of which
4 is 4 inches. Show by means of diagrams the position and magnitude of the image in each case.

6 **6.** (*a*) Describe an experiment to demonstrate the presence and direction of the lines of force in the magnetic field of a bar magnet.

(*b*) Show by means of an experiment that a magnetic field
4 surrounds (1) the earth, (2) a conductor through
4 which an electric current is passing.

4 (*c*) State the conditions which determine the strength of an electro-magnet.

6 **7.** (*a*) Describe an experiment to show the condition necessary for the production of an induced current in a closed circuit.

4 (*b*) How has this condition been fulfilled in the dynamo and induction coil?

9 (*c*) What are the essential parts of a direct current dynamo and the function of each part?

8 **8.** (*a*) How may electric waves be generated and detected?
10 (*b*) Describe the construction and action of the receiving apparatus for wireless telegraphy.

125

STATE PHYSICS EXAMINATION.

Interscholastic High School Contest Held at Norman, Okla., May 6, 1921.

Instructions: In the upper right hand corner of each sheet write your number. Do not in any way indicate your name or school, answer eight questions, not more than three from a group. Short answers that cover the ground are better than long ones.

MECHANICS.

1. A man aims his gun at a monkey in a tree 150 yards away by sighting through the barrel. The gun is fired and the monkey, seeing the flash, immediately falls to the ground. Does the monkey get hit? Give reasons for your answer.
2. A man working on the edge of a roof 200 feet above the sidewalk sees a heavy plank sliding over the edge and shouts a warning to the street below. If the time taken by his voice be 18 seconds, how much time will elapse between the arrival of his warning and the impact of the plank?
3. A man is on an elevator. In each of the following cases indicate whether or not the motion of the elevator affects the apparent weight of the man on the elevator (that is, the force with which he presses on the elevator floor) and if it does whether the apparent weight is greater or less? (1) Moving upwards with uniform speed, (2) moving upwards with increasing speed, (3) moving upwards with decreasing speed.
4. A bomb is to be dropped from an airplane flying at an altitude of 500 feet at 60 miles per hour in still air. If the bomb is to hit the target how many feet (horizontal distance) must the airplane be from the target when the bomb is released.

HEAT.

5. Why does a fan give relief from the oppressive heat of a hot summer day? Why does a wind aid in the drying of wet clothes?
6. Which is the more effective, a tea kettle with a polished bottom or one with a blackened bottom and why? Why are the walls of a thermos bottle silvered?

7. Draw simple diagrams showing the four stages in the complete "cycle" of a 4 cycle, single cylinder, gas engine, telling by a sentence under each what is going on.

SOUND.

8. Will the pitch of an organ pipe be higher or lower on a cold day than a hot day? Explain.

ELECTRICITY.

9. An electric iron drawing 5 amperes on a 110-volt circuit is used for 4 hours. How much will it cost if the power charge be 10c. per kilowatt-hour?
10. Describe the operation of a dynamo.
11. Why is the bird cage at a zoo the safest place to be in a thunder storm?
12. Two resistances of 4 and 6 ohms respectively are connected in series and one of 12 ohms is shunted across them. What is the total resistance of the combination? What voltage will send 2.2 amperes through this system?

LIGHT.

13. "We see the heavens as they were, not as they are." Explain.
14. An airplane carries a camera with axis vertical. The lens has a focal length of 12 inches. A picture is taken when the airplane is at an altitude of 1,000 feet. A building 100 feet long will appear how long in the picture?

COLLEGE ENTRANCE EXAMINATION BOARD.

PHYSICS.

Friday, June 24, 1921, 9 a. m. Two hours.

Answer ten questions as indicated below. No extra credit will be given for answers to more than ten questions.

Indicate clearly your reasoning in each problem and state the units in which each answer is expressed.

Number and letter each answer to correspond to the questions selected.

GROUP I.

(Omit one question from this group.)

1. (*a*) Describe the construction and explain the operation of an aneroid barometer.
   (*b*) Explain how the height attained by an aeroplane may be determined by means of such an instrument.
2. What fractional part of the air originally in the receiver of an air pump, when open to normal atmospheric pressure, still remains when the pressure within the receiver is equivalent to only 1.2 centimeters of mercury?
3. (*a*) A boy is able to exert a force of 75 pounds. What is the shortest plank that he might use to enable him to push a truck weighing 300 pounds along the plank up to a doorway 3 feet above the ground?
   (*b*) How much work would be done in raising the truck?
4. On the Isle of Man is an overshot water wheel of 150 horse-power having a diameter of 72.5 feet, a width of 10 feet, and an efficiency of 85%. How many cubic feet of water pass over the wheel each second?

GROUP II.

(Omit one question from this group.)

5. Describe an experimental method by which the velocity of sound may be directly determined in the open air.
6. A blow struck with a hammer on a steel cable was heard through the cable in 0.2 second and through the air 2.8 seconds later. The temperature was 18° C.
   (*a*) How far away was the blow struck?
   (*b*) What was the velocity of sound in the cable?

GROUP III.

(Omit one question from this group.)

7. (*a*) Drinking-water is sometimes cooled by placing it in a jar of porous earthenware. Explain.
   (*b*) What prevents a pond from freezing solid in winter when the temperature of the air is below 0° C.?
   (*c*) State the different means by which heat may be transmitted from one place to another, giving examples of each method.
8. If the span of a transmission line is 800 feet long at 0° C. and 800.704 feet long at 40° C. what is the coefficient of expansion of the wire?
9. A glass tumbler weighing 150 grams contains 250 grams of water at temperature 20° C. How much ice at temperature 0° C. should be introduced to lower the temperature of the tumbler of water to 10° C. assuming that all of the ice is melted? The specific heat of glass = 0.18.

GROUP IV.

(Omit one question from this group.)

10. If 4 foot-candles is the proper illumination for reading, how far from the page of a book should a 16-candle-power light be placed in order to give this illumination? (A foot-candle is the illumination given by one candle 1 foot away.)
11. Describe the characteristic changes in his image which may be observed by a man as from a distance he approaches in turn
   (*a*) a plane mirror,
   (*b*) a convex mirror,
   (*c*) a concave mirror.
12. (*a*) Explain the meaning of the term index of refraction.
   (*b*) Assuming the index of refraction for light passing from air to glass to be 1.6, determine the speed of light in glass.

GROUP V.

(Omit one question from this group.)

13. (*a*) What are the essential parts of an electromagnet?
   (*b*) In what two ways may the strength of an electromagnet be increased?
   (*c*) State three respects in which an electromagnet is more advantageous than a permanent magnet.
   (*d*) In navigation why must corrections be applied to the readings of the magnetic compass?
14. If 6.7 grams of silver are deposited on the cathode in an electrolytic plant in one minute when a current of 100 amperes is used what is the electro-chemical equivalent of silver?
15. At a cost of 4 cents per kilowatt hour how much will it cost per week of 48 hours to run a motor having an average load of 35 horse-power and an average efficiency of 85%? One horse-power = 746 watts.

SENEGAL: AN ADVANCED POST OF AFRICAN MOHAMMEDANISM.

"Senegal, brought into prominence because of the thousands of Senegalese who as soldiers or laborers assisted the French in the World War, is the gateway and main trade route to France's vast West African possessions," says a bulletin issued from the Washington, D. C., headquarters of the National Geographic Society.

"In Senegal, and in the regions inland to the east, Mohammedanism holds almost undisputed sway; and from there, some observers declare, there is likelihood that this religion will sweep southward until the great majority of African natives will become Moslems in name at least.

"On its west or Atlantic coast, Senegal may be said to mark the beginning of the real Africa—the hot, moist 'dark continent' peopled with negroes. For northern Africa is to a large extent desert, and is the land of

Arab and Berber or 'Moor.' As one steams down the Atlantic coast of Africa, leaving the last of the perennial streams in Morocco, he skirts a forbidding desert coast without a permanent stream for 1,800 miles. The first large stream encountered is the Senegal, emptying into the sea through a system of lakes and lagoons, navigable during the rainy season for 600 miles inland. This was naturally one of the best trade routes into the interior of northwestern Africa, and by the contruction of a railroad from the river's rapids to a point 435 miles farther inland, it has been made still better.

SENEGAL HAS DEPUTY IN FRENCH HOUSE.

"The foothold of France in Senegal dates from the days of Richelieu. In 1636 Fort St. Louis was established as a trading post at the mouth of the Senegal River. France could point to little more than the maintenance of coast stations in this region, however, until relatively recent times. It was not until near the middle of the nineteenth century that there was anything like an aggressive effort to consolidate Senegal under French control. How great the progress has been since that time is indicated by the fact that there are now four self-governing municipal communes in Senegal, and they jointly send a deputy to the French Chamber in Paris. The portion of Senegal which may be regarded as having progressed far toward becoming, like Algeria, a part of France, is small, embracing less than 1,000 square miles and having a population of only a few hundred thousands. The remainder of Senegal, with an area of more than 70,000 square miles, consists of native states under French protection and under the close supervision of resident agents.

"Situated between desert Africa and moist tropical Africa, Senegal is not unnaturally a region of mixed races. As one advances from the north, the Senegal River marks the first encounter with predominantly negro tribes, as it marks the end of the desert and its fringe of semi-desert. On the north side of the river dwell people of Berber or 'Moorish' stock—swarthy, straight-haired, straight-nosed. To the south of the river the tribes vary much in appearance but practically all are negroes—black or dark chocolate colored, with broad flat noses and kinky hair. Close to the coast are the members of the Wolof tribe who are usually regarded as typically Senegalese. They are said to be the blackest negroes known. Even their lips are black. The people of this tribe are particularly loyal to the French and have served in large numbers as soldiers.

EXAMPLE OF "PACIFIC PENETRATION."

"In administering Senegal, as indeed in administering all her African possessions, France has depended on tact rather than on force. Her officials not only recognize native chiefs, but also treat them with consideration. The greatest of the rulers are presented with valuable gifts which emphasize the wealth and power of France, and some of the leaders are taken to Paris as guests of the government. The French administrators have earned the gratitude of the large and influential trading class in Africa by breaking up brigandage and policing the caravan routes. The Moslems are all grateful because of the French policy, which has insured privacy to the mosques and harems and has permitted them to continue their schools and colleges and in some cases has even subsidized such institutions.

"But though France has permitted Moslem schools to be carried on, she has not left all schooling to the Mohammedans. Public schools have been established in Senegal as elsewhere in the West African possessions, in which children of the various races, tribes, and religions are taught in French. At present these schools reach only a few thou-

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sand children in Senegal, but the system is being extended as rapidly as possible.

"Excellent roads have been built by the French in Senegal, connecting the various towns and supplementing the river and rail routes to the seacoast. Dakar, which is situated under the lee of Cape Verde, the westernmost point of land in Africa, is the best port on the whole west coast, and almost the only port on that coast at which ocean-going vessels can tie up alongside a quay protected from the surf. As a result, the commerce of the port is heavy, and the trade route to the east starting there is used to distribute and collect supplies to and from extensive regions in the interior. Dakar is the seat of administration for all French West Africa and has a larger European population (about 3,000) than any other town in Senegal or the neighboring countries.

NOT A "WHITE MAN'S COUNTRY."

"Senegal is not a 'white man's country.' Because of its climatic conditions it is unhealthful for Europeans and probably always will be. The French recognize this and have made no efforts to colonize the country. The European population is made up almost entirely of administrators of various kinds, and traders. This situation has made necessary the French plan of organizing the country largely on the basis of native states under protection.

"Although cattle-raising on the grassy plains of the upper Senegal has long been the chief industry, agriculture has attained a place of greater importance in recent years. The principal article of export is the peanut, and considerable amounts of other oil seeds, oils, hides, rubber, timber, and cocoa also are sent out. Senegal ships large quantities of kola or cola nuts. These are eaten by the natives throughout western Africa and even in the more remote parts of the continent, because of their stimulating qualities.

"Cotton grows wild and its production might be made into an important Senegalese industry but for the habits of the natives. They are unwilling to engage in the cultivation of a crop which requires considerable steady work and a relatively long wait for maturity. They prefer to raise such crops as peanuts and millet which require little attention between planting and harvest.

SMALL STEP TO MOHAMMEDANISM.

"One way of understanding the religious situation in Senegal is to consider that the inhabitants are at the level or much below the level of civilization of the tribes of northern Europe before their contact with the Roman Empire. The Senegalese have had little opportunity to adopt Christianity, but even so their conditions and surroundings predispose them to an acceptance of Mohammedanism since its principles as practiced in Mohammedan Africa more closely accord to their degree of culture. They do not find it easy to rise to the conception of a loving, just, merciful God; but, used to tyranny and the exercise of arbitrary power, they more easily accept what has come to be the Mohammedan conception of Allah. Of even more importance to them, they can become Mohammedans without changing their fundamental customs. In particular they may retain the polygamy which they already practice, and outside immediate French control may hold slaves.

"Not of least importance in the spread of Mohammedanism in Senegal is the fact that negroes are instrumental in extending it. Not unnaturally the natives are inclined to give ear to men of their own race living their own lives, as against Europeans who live differently and would have their converts live differently.

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"Whether the Moslem religion will obtain a firmer or a less firm hold on Senegal and the remainder of 'black Africa' is believed by some observers to depend largely on the language question. Arabic is being extended as the language of trade, and it is the language of the Koran If France insists on the use of the French language in all intercourse with the natives, as she is showing a tendency to do, and teaches it to a large proportion of the children in the public schools, the spread of Mohammedanism, it is believed, will be much less easy."

ELECTRIC POWER IN THE UNITED STATES.

STATEMENT BY U. S. GEOLOGICAL SURVEY.

An excellent index of industrial conditions throughout the United States is furnished by statistics of the production of electricity for power and light by the public-utility companies. In times of depression less electric power is used by manufacturers who purchase electricity, and the electric railways have less traffic and therefore call for less power. Conversely, in times of prosperity the demand for electricity is greater from every source, even for electric light, in using which the consumer feels less need of economy.

For more than two years monthly statistics have been obtained by the United States Geological Survey, Department of the Interior, from over 3,000 public-utility electric plants, which represent from 90 to 95 per cent of all such plants in the United States. These generating stations include central stations, electric-railway plants, and certain other industrial concerns which produce electricity for public use in addition to that needed for their own requirements. These statistics are compiled for each state and are published monthly in mimeographed form in sufficient detail to be used as a basis for many economic studies.

The statistics compiled show the number of kilowatt-hours produced monthly by water power as well as the number of kilowatt-hours produced by all fuels. The quantities of fuel of different kinds (coal, fuel oil, and natural gas) consumed monthly in producing electric power by these companies are also given.

The monthly reports give these statistics not only for the current month but for a number of preceding months, so that comparisons may readily be made. They also contain curves showing the average daily production of electricity by these companies both by water power and by fuels.

On May 12, 1921, a 14-page summary report for the year 1920 was issued in mimeograph form giving not only the class of statistics mentioned above but also the detailed percentage produced in each state of the total power produced in the United States and the percentage of each state's total production that was made by water power and by fuels.

Comparative curves of the average daily production of electricity by public-utility plants for the complete years 1919 and 1920 are also shown, as well as a statement calling attention to some of the outstanding facts brought out by the data.

Copies of the monthly and summary annual reports mentioned may be obtained from the Director, United States Geological Survey, Washington, D. C.

WORLD'S LARGEST DEPOSIT OF ROCK SALT.

Vast quantities of rock salt lie less than half a mile beneath the surface of the earth in the United States. In New York, Ohio, Michigan, Penn-

We Cover the Earth

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Every State in the United States, every Province in Canada, Mexico, Cuba, Porto Rico, Brazil, Argentine, Chile, Peru, Ecuador; nearly every country in Europe (before the war every country); Egypt, Liberia, Cape Colony, The Transvaal, Persia, Ceylon, India, China, Korea, Japan, Philippines, New Zealand, Australia and Hawaii.

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sylvania, West Virginia, and other states there are large deposits, but the largest deposit in the United States, and probably in the world, is that which extends from northern Kansas across the west end of Oklahoma, the pan-handle of Texas, and southeastern New Mexico to western Texas. The area underlain by these great Permian salt deposits is not far from 100,000 square miles, according to the U. S. Geological Survey, Department of the Interior. The limits of the deposit, especially to the northwest and southeast, have not been ascertained, but in general the area of thick salt extends fully 650 miles from northeast to southwest, and is 50 to 150 miles wide. The thickness and the succession of the beds are variable, but 700 feet is reported in one hole, and in many places the aggregate is more than 300 feet. On the assumption of an average thickness of 200 feet of salt, the gross quantity in the area of 100,000 square miles is so large, about 30,000 billion tons, that the present needs of the United States (about 7,000,000 tons a year) can be supplied for millions of years.

A map of this great deposit and a statement of the evidence of its existence are given in Bulletin 715-M, entitled, "Permian salt deposits of the south-central United States," by N. H. Darton, which has just been published and can be obtained from the Director, United States Geological Survey, Washington, D. C.

GRINDSTONES AND PULPSTONES.

INCREASE IN SALES IN 1920—ACCORDING TO UNITED STATES GEOLOGICAL SURVEY.

According to reports submitted to the United States Geological Survey, Department of the Interior, 44,832 tons of grindstones, valued at $1,239,990, was sold in 1920. The sales in 1919 amounted to 40,755 tons, valued at $993,959. This material was produced from sandstone quarries in Michigan, Ohio, and West Virginia. The sizes of the grindstones vary greatly and are reported as square grindstones, from ¼ inch to 8 inches in diameter, as lathe stones, from 6 to 12 inches in diameter, and as grindstones, from less than a foot in diameter and less than a pound in weight to large stones 6 to 7 feet in diameter, 8 inches to a foot or more in thickness, and from 1 to 2 tons in weight. The prices of the stones differ according to the size and quality of the stone. The smaller sized stones which are sold by the piece, show a much higher average value per ton than the large stones. The ordinary sizes were reported to range from $23 to $40 per ton f. o. b. (unmounted) at the quarry. The average value per short ton, of all the grindstone material sold in 1920 was $27.66; the average in 1919 was $24.39.

Pulpstones are heavy stones used for grinding wood into fine fiber for making pulp and paper. These stones are generally 54 inches in diameter and 27 inches thick and weigh about 2 tons but some machines require stones 62 inches in diameter, 54 inches thick, and that weigh about 4 tons. The production in 1920 was 8,652 tons, valued at $467,014, and in 1919 it was 6,110 tons, valued at $342,056. The average value per ton was $54 in 1920 and $56 in 1919. Pulpstones were obtained from quarries in Ohio and West Virginia.

Business conditions in this industry were unsettled throughout the year, and though the demand was good the sales were lessened by difficulties with transportation and with labor.

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SECRETARY OF THE MATHEMATICAL ASSOCIATION
OBERLIN, OHIO

THE ORIGIN OF LONG ISLAND.

Long Island affords particularly clear evidence as to the history of the great continental ice sheet which covered the Northern States many thousand years ago.

The southern margin of this great ice sheet, according to the United States Geological Survey, Department of the Interior, extended to Long Island and remained there for a long time, depositing a thick body of intermixed boulders, sand, and clay as a terminal moraine, which is now the "backbone" of the island. The ice moved southward across the area that is now occupied by Long Island Sound and brought these materials from the north, dropping them at its melting edge. This method of deposition developed a very peculiar topography, consisting of an irregular aggregation of hummocks and hollows, which have produced the many beautiful details of configuration that make the higher parts of Long Island so attractive to lovers of nature. The most notable of these hollows in the morainal ridge is the one holding the picturesque Lake Ronkonkoma, which lies in a depression 50 feet below the surrounding ridges. Several other similar pits are 80 to 85 feet deep, and some of the larger irregular hollows are several miles long.

The older rocks of Long Island consist of the granite floor, which comes to the surface near East River and Hell Gate, and clays and sands of Cretaceous age, which not only crop out at places along the north shore but rise into high mounds that lie under the surface deposits at places in the west-central part of the island. The geologic history of Long Island is a long and complicated story, but many of its details are clearly indicated by the character and the relations of the various deposits and by the diversities of configuration.

Madison, Wis., August 22, 1921.

To Members of the Central Association of Science and Mathematics Teachers:

Let me urge you to renew your membership in the Association at once. The treasurer, Mr. L. L. Hall, 11043 Hermosa Avenue, Chicago, is authorized to receive $2.55 (including 5 cents exchange on personal checks) as dues for 1921-1922.

Have you persuaded some one else to join the association for the year 1921-1922. If not, do it now!

Watch the November number of SCHOOL SCIENCE AND MATHEMATICS for the announcements concerning the meeting at St. Louis. If you can do so, plan to be at the meeting at St. Louis.

For the Central Association,

WALTER W. HART,
President.

AMENDMENT TO THE CENTRAL'S CONSTITUTION.

We recommend an amendment to the constitution to the effect that the dues be two dollars and fifty cents ($2.50) per year.

CALIFORNIA INSTITUTE OF TECHNOLOGY.

Dr. Robert A. Millikan, professor of physics at the University of Chicago, who has gained world-wide distinction for his original researches on the electron and the structure of matter, has been appointed director of the Norman Bridge Laboratory of Physics at the California Institute of Technology and Chairman of the Executive Council of the institute. Dr. Millikan has accepted the post and will come to Pasadena in October,

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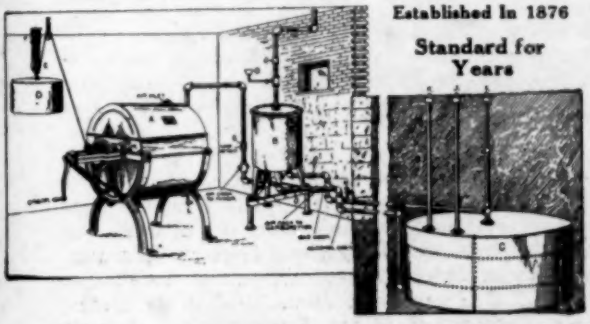

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at which time the new Norman Bridge Laboratory of Physics will be ready for occupancy.

In associating himself permanently with the California Institute of Technology, Dr. Millikan plans to devote himself mainly to the development here of one of the most important centers of research in the country, and it is the desire of the trustees that he be free to do this without being burdened with administrative duties.

When the Norman Bridge Laboratory of Physics is fully in operation an annual appropriation for its support of approximately $100,000 has been guaranteed. This fund will make possible the securing of a large staff of the ablest investigators and teachers, and unusually complete equipment.

In addition to this provision for annual support, the institute has recently received from Dr. Bridge the promise of $200,000 for an extension of the laboratory and of $50,000 for its library.

Dr. Millikan was graduated from Oberlin College in 1891. He received the degree of A. M. from Oberlin in 1892, and the degree of Doctor of Philosophy from Columbia University in 1895. He was elected to both Phi Beta Kappa and Sigma Chi. He studied at Berlin and Gottingen during 1895 and 1896, and became a member of the staff of the physics department of the University of Chicago in the latter year. He has since been given the honorary degree of Doctor of Science in recognition of his scientific attainments by Oberlin College, Northwestern University, the University of Pennsylvania, Columbia University and Amherst College.

In 1913, Dr. Millikan was awarded the Comstock prize by the National Academy of Sciences for his researches in electricity. He was the Hitchcock lecturer at the University of California in 1917, and Clark lecturer at Amherst in the same year. During the war Dr. Millikan served as vice chairman of the National Research Council, as lieutenant-colonel in the Signal Corps, and chief of the science and research division of the Signal Corps. He is a trustee of Oberlin College.

Dr. Millikan is foreign secretary of the National Academy of Sciences and a member of the American Philosophical Society, and he has been named honorary member of the Royal Institution (London) and of the Society Batave de Philosophie Experimentelle of Rotterdam. He was President of the American Physical Society from 1916 to 1918, and is now President of the American association of University Professors. He is the author of text-books on physics, which are extensively used; of a book, "The Electron," and of numerous scientific papers. Dr. Millikan has had a part-time connection with the California Institute of Technology since 1917. He was recently chosen to make the address at the National Museum in Washington following the presentation of a gram of radium to Mme. Curie. He spoke on "The Significance of Radium" and his address is shortly to be published by the California Institute of Technology.

PHOSPHATE ROCK IN MONTANA.

Interest in the possibility of mining phosphate in the West has been revived during the last two or three years, and more or less work has been done in the Garrison, Melrose, and Philipsburg fields in Montana. Phosphate deposits near Maxville, Granite County, Mont., in the Philipsburg field, are described by T. J. Pardee in Bulletin 715-J of the United States Geological Survey, Department of the Interior. The deposits contain a large quantity of easily accessible phosphate within a few miles of the railway. The bulletin, which contains a map and describes the deposits, may be had without charge from the Director of the United States Geological Survey, Washington, D. C.

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RECEPTION TO THE MEMBERS OF THE NATIONAL COMMITTEE ON MATHEMATICS REQUIREMENTS FOR SECONDARY SCHOOLS.

By Rev. W. J. Ryan,
St. Louis, Mo.

Friday, April 22, 1921, will long be remembered by every teacher of mathematics in St. Louis and immediate vicinity. On that evening the Mathematics Club of St. Louis and vicinity tendered a banquet to the Members of the National Committee on Mathematics Requirements for Secondary Schools. The committee had held a meeting on Thursday, April 21, at the Statler Hotel.

An informal reception was held from 6:30 p. m. to about 7:30 p. m. in the lobby of the American Annex Hotel. The guests quickly filled all available space and when the time appointed for sitting down arrived, the large dining room was filled to crowding. Almost two hundred fifty were in attendance. They came not alone from the ranks of the mathematics teachers, nor even science teachers, but the principals of public and many private schools, the Assistant Superintendents of Education of the City of St. Louis, and men and women interested in education were those who gathered to enjoy the tasty meal for which Misses Sherry and Long had arranged so carefully. The decorations were simple and in excellent taste and drew many pretty compliments from the banqueters. At the speakers' table were President Alfred Davis of the St. Louis Mathematics Club; Dr. John J. Maddox, Acting Superintendent of Education in St. Louis; Dr. David Eugene Smith, President Robison of St. Louis University, Prof. Tyler, Prof. Young, Prof. Crathorne, Prof. George R. Throop, Vice-Chancellor of Washington University, and Rev. W. J. Ryan, Professor of Mathematics at St. Louis University. The members of the National Committee chose to sit at different tables and come to know the guests of the evening.

President Davis opened the speaking of the gathering and told of the efforts of the Mathematics Club to place mathematics in its proper position in the secondary schools. The only defect in his talk was the failure to make clear to all how splendid and tireless he has been in bringing the Mathematics Club from zero efficiency to its present commdaning position.

Dr. Maddox was next introduced as the toastmaster of the evening and proved his fitness for the position to the satisfaction of all as the evening lengthened. His manner was pleasant, his introductions brief and his hearty approval of the affair made for much in the cause of mathematics in St. Louis. President Robison of St. Louis delivered a very impressive speech on education, deprecating the trend of the present day in some who seek to turn out money-making machines, and forget the true nobility of man. He then laid stress upon the need of mathematics in the secondary schools, showing how it develops reasoning power, grasp of general conclusions, love of research and independence of thought. Miss Eula Weeks, our local member of the committee gave a charming talk and modestly declined to say aught of herself in mentioning the good work being accomplished by the Mathematics Club. Prof. George A. Throop, Vice-Chancellor of Washington University, gave a splendid talk on the relation of the high school to the college. He deprecated sincerely the growing tendency to push the hard and worthwhile studies, such as mathematics, out of the course of high school subjects and place them in the colleges. He also noted that those who had profited by early training on mathematics generally succeeded in the college subjects. In conclusion he vigorously urged the making of

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mathematics obligatory for some years, at least, in every High school course. Prof. J. W. Young, of Dartmouth, Chairman of the National Committee, made a neat address and emphasized the need of leisure for the teacher of mathematics, insisting that the day a teacher ceased to be a student he or she also ceased to be a real teacher. Miss Vevia Blair, of Horace Mann School, New York City, gave some very interesting statistics upon mathematics in its relation to the transference of training. Mr. Raleigh Schorling of the Lincoln School, New York, briefly and humorously told of the place of mathematics in the lives of boys and girls and was followed by Mr. Downey of Boston who dwelt upon the need of making the subject attractive to the pupils and thus inspiring interest. Prof. C. N. Moore told very interestingly of the value of mathematics in everyday life. He showed with quite definite statistics the dependence of civilization upon the results which mathematics has unsealed to the world in general.

Prof. David Eugene Smith was the chief speaker of the evening and was given the last position on the program. His address on mathematics and citizenship was inspiring, his humor catching, his scholarship manifested itself in his grasp of human life and the expression of his thought. For more than twenty minutes he held riveted upon him the close attention of every individual present and the effect for the furtherance of mathematics in St. Louis Schools will be increasingly felt as the years run on. President Davis then suggested that the banqueters should mingle in a social way and his suggestion was taken up at once. The whole affair was kept on a high level from the opening remarks until the close of Prof. Smith's wonderful address. Many were outspoken in their happiness at being present and each individual seemed to realize that he or she had spent a pleasant evening and a vastly profitable one as well. The banquet of the St. Louis Mathematics Club marked a new epoch in the advancement not alone of mathematics but of education in general. It was an especially timely affair for the high schools of St. Louis. The trend seemed to have been away from mathematics; it was being gradually pushed aside from the important place it has ever occupied in every truly educational system. The presence and active cooperation of Superintendent Maddox as well as of the principals of the high schools would seem to augur well for the complete restoration of mathematics to a lasting place in the curriculum of studies in the public and private schools of St. Louis and vicinity. Much credit for the success of the banquet is due to President Davis, to Miss Meta Eitzen, Secretary-Treasurer, and to Miss Weeks and Mr. Ammerman, members of the Executive Committee of the Mathematics Club. We feel sure that the members of the National Committee returned to their various homes inspired with the new zeal for the cause of mathematics and profound regard for the work accomplished in St. Louis by the members of the Mathematics Club of St. Louis and Vicinity.

PROGRESS OF MINING IN ALASKA.

To provide fuel for the use of the Alaskan Engineering Commission coal mining was continued in 1919 in the Matanuska field, Alaska, on about the same scale as in previous years. Near this field and also tributary to the Government railroad, is the Willow Creek gold district, where large auriferous lodes are being exploited. These mining developments are described in a pamphlet by Theodore Chapin entitled "Mining in the Matanuska Coal Fields and the Willow Creek District, Alaska," just issued by the U. S. Geological Survey, Department of the Interior, as Bulletin 714-D.

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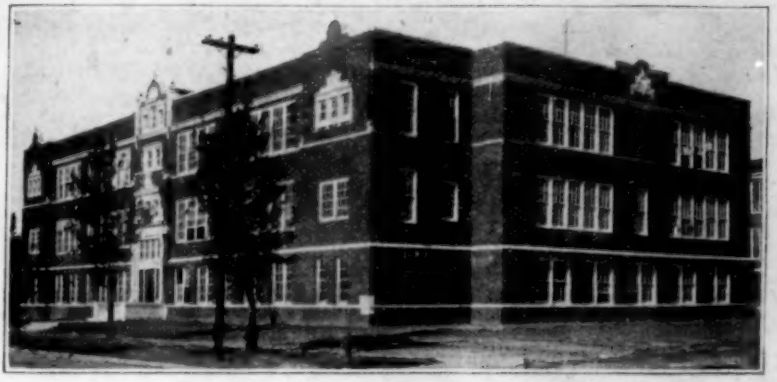

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PERSONALS.

The editor of SCHOOL SCIENCE AND MATHEMATICS was in charge, during the past summer, of the high school at Camp Roosevelt, Muskegon, Michigan. This is the largest boys' camp in the country.

Professor Philip B. Woodworth, for many years head of the physics department of Lewis Institute, Chicago, has recently been elected President of the Rose Polytechnic Institute at Terre Haute, Indiana.

Dr. W. E. Stone, President of Purdue University, LaFayette, Indiana, was killed by falling into a crevasse of a glacier on Mount Eanon, British Columbia, the past summer.

Mr. Lewis L. Hall, Treasurer of the Central Association of Science and Mathematics Teachers and teacher of chemistry and physics in Morgan Park High School, Chicago, taught his subjects in the summer high school at Camp Roosevelt, during July and August.

Mr. Irvin A. Wilson, principal of the Sherwood School, Chicago, Mr. G. C. Staley, of the Parker High School, Chicago, and Mr. William J. Page, Marshall High School, Chicago, all taught mathematics in the Camp Roosevelt summer high school, Muskegon, Mich., this summer.

Mr. Ira N. Van Hise, of the Hyde Park High School, Chicago, had charge of the General Science and Commercial Geography work at Camp Roosevelt, during the summer recess.

Professor Hale Houston, of the civil engineering department at Clemson College, has been made professor of engineering at Washington and Lee University.

The Bureau of Standards suffered a great loss in the death of Dr. Edmond B. Rosa, its chief physicist.

Clark University has received a fine addition to its faculty by the appointment of Dr. Charles T. Books of the Weather Bureau to the chair of meteorology.

"Dr. Henry Platt Cushing, Cornell '83, for thirty years professor of geology in Western Reserve University, Cleveland, and for a long time geologist in the Adirondack region for the Geological Survey of New York, died on April 14."

Mr. V. O. Graham of the Lake View High School, Chicago, this past summer conducted very interesting classes in botany and Zoology at Camp Roosevelt, Muskegon, Mich. Mr. Graham has identified over 700 plants in the region near the camp.

Dr. Frank C. Baker, of the University of Illinois and a frequent contributor to this journal, spent the summer in Wisconsin studying the molluscan fauna of the state.

Professor R. A. Dutcher, of the University of Minnesota, has been elected head of the department of chemistry in the College of Agriculture, Pennsylvania State College.

Howard M. Raymond, for many years dean of Commerce Institute Chicago, has been elected to the presidency of that institution.

Professor Ira M. Howley, of Cornell University, has recently been appointed to the chair of zoology in the Utah Agricultural College.

Professor L. K. Richtmyer, of the department of physics, of Cornell University, has been giving, at Schenectady, under the direction of the General Electric Company, a course of lectures on modern physical theories.

Dr. Robert A. Millikan, of the University of Chicago, recently delivered the first annual address before the Crowell Scientific Society of Trinity College, Durham, N. C.

Dr. Albert C. Hole, the well-known science teacher of Brooklyn, for

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thirty years teacher of the physical sciences in the Boys' High School, Brooklyn, died April 22. He was seventy-five years old.

Dr. James Law, the first director, and, since his retirement a few years ago, director emeritus of the New York State Veterinary College, Cornell University, died on May 11, aged 83 years.

Dr. Robert C. Duncan, a physicist of the Bureau of Standards has been appointed to the position of technical expert in the Bureau of Ordnance.

Dr. Millard S. Markle, research editor in biology of this journal, and professor of Botany in Earlham College, taught in the Cornell University summer school the past season.

Dr. John C. Hessler, for some time a member of the Board of Administration of Mellers Institute of Industrial Research, of the University of Pittsburgh, has accepted the position of head of the department of Chemistry in Knox College, Galesburg, Ill.

ARTICLES IN CURRENT PERIODICALS.

American Forestry, for June; *1214 16th Street, Washington, D. C.* "Your National Parks," Stephen Mather; "Unfamiliar Scenes in National Parks," Herbert W. Gleason, eighteen illustrations; "Stephen T. Mather—the Man Who Did It," one illustration; "The Value of Our National Parks," Albert B. Fall; one illustration; "Our National Parks and How to Reach Them," Arthur E. Demaray, fourteen illustrations; "Uncle Sam's Most Northern Woodlot," L. C. Pratt, twelve illustrations; "Reforestation in the Middle States," William E. Hays; five illustrations; "Washington's Forest Catastrophe," Heister D. Guie, five illustrations.

Condor, for May-June; *Eagle Rock, Los Angeles Co., California*; $2.00 *per year*, 40 *cents a copy:* "Notes on the Nesting of the Yosemite Fox Sparrow, Calliope Hummingbird and Western Wood Pewee at Lake Tahoe, California" (with one photo), John W. Maillard; "The Probable Status of the Pacific Coast Skuas," A. C. Bent; "Nesting of the Stephens Fox Sparrow" (with three photos), Wright M. Pierce; "Weights and Plumage of Ducks in the Rio Grande Valley," Aldo Leopold; "A Bird Census at Prescott, Walla Walla County, Washington" (with one photo), Lee R. Dice; "Additional Notes on the Water and Shore Birds of Netarts Bay, Oregon" (with two photos by O. J. Murie), Stanley G. Jewett; "New Bird Records for North America, with Notes on the Pribilof Island List," Joseph Maillard and G. Dallas Hanna.

General Science Quarterly, for May; *Salem, Mass.* $1.50 *per year*, 40 *cents a copy:* "Method of Vitalizing the Study and Teaching of General Science," J. Richard Lunt; "General Science in Minnesota: Outline of Course," Committee; "The Pupil's Interest as a Foundation in Science Teaching," M. C. Collister; "Standardizing of First Year Science Tests," P. A. Maxwell; "Five Years of Fire Waste," "Artificial Lighting as Compared to Natural Lighting," J. H. Kurlander; "Dental Service for Rural Schools," W. A. Brierley; "The Nostrum and the Public Health," A. J. Cramp; "Hygiene Projects for the Upper Grades."

Journal of Geography, for May; *Menasha, Wis.* $2.00 *per year*, 25 *cents a copy:* In this issue—"Agricultural Meteorology," J. Warren Smith; "Some Geographic Factors in the Development of Chicago," Fred K. Branom; "Forty-eight States and Their Population," Douglas C. Ridgley; "A Geographic Magazine as a Class Project," Elizabeth Wolff.

National Geographic Magazine, for July; Washington, D. C.; $4.00 *per year*, 50 *cents a copy:* "Life on the Grand Banks," 29 illustrations, Frederick W. Wallace; "Scenes from France," 16 illustrations; "The Geography of Japan," 39 illustrations, Walter Weston; "The National Geographic Society Completes Its Gifts of Big Trees;" "Adventures with a Camera in Many Lands," 24 illustrations, Maynard O. Williams.

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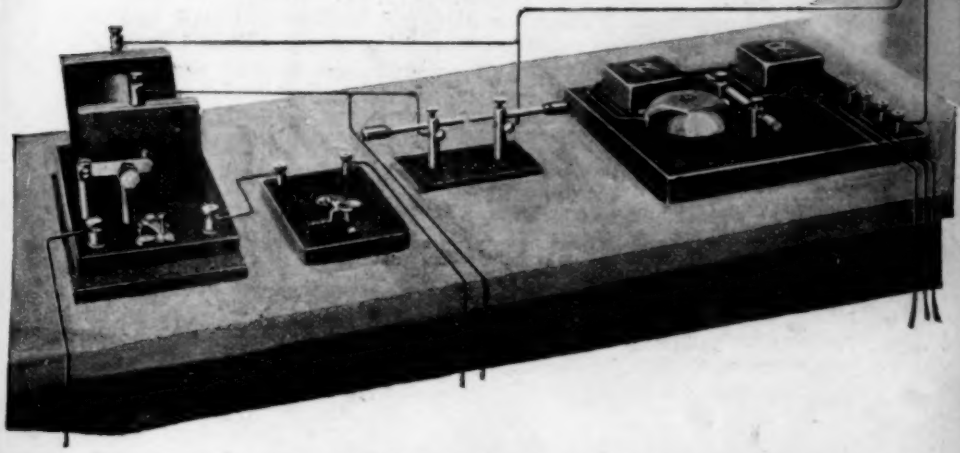

Nature-Study Review, for April; *Ithaca, New York* $1.50 *per year*, 20 *cents a copy:* "First Nature Study School," W. G. Vinal; "Where Should Nature Study Come on the Program?" Mrs. Charlotte Gulick; "Counsel for Councillors," W. G. Vinal; "Learning to See and Hear," Agathe Deming; "Nature Work at Camp Kehonka," Virginia F. Birdsall; "Nature Study at Highland Nature Camp," Allen B. Doggett; "Nature at Teela-Wooket," Mary A. Stillman; "Nature-Study in Summer Camps for Girls," Eleazer J. Dole; "Nature Study, Normal School and Summer Camp Councillors," Grace B. Gardner.

Photo-Era Magazine, for June; *Boston, Mass.* $2.50 *per year*, 25 *cents a copy:* "The Book of Nature," Frederick B. Hodges; "Getting More Business by Using Displays," Frank H. Williams; "Portraiture at the Camera Club," Floyd Vail, F. R. P. S.; "An Experience in Low-Power Micro-Photography," Peter W. Saul; "An Attempted Photographic 'Come-Back,'" Charles W. Long; "Pinatypy," R. Wagner.

Popular Astronomy, for June-July, *Northfield, Minn.* $4.00 *per year:* "Sherburne Wesley Burnham," with Plate XV., E. E. Barnard; "The Position of the Sun's Axis," Frederick R. Honey; "The Parallactic Motion of the Stars in Zones, R. H. Tucker; "Interesting Facts About Our Own Planet," Charles Nevers Holmes; "The 'Quantum' Equation in the Solar System," Fernando Sanford; "The International Date Line," Roscoe Lamont.

School Review, for June; *University of Chicago Press* $2.50 *per year*, 30 *cents a copy:* "Where to Establish Junior Colleges," Leonard V. Koos; "Guidance in the High School with Special Reference to College Entrance," John M. Brewer; "The North Central Association," C. O. Davis; "Some Standards for Home Project Work in Agriculture," W. P. Dyer.

Scientific Monthly, for July; *Garrison, New York* $5.00 *per year*, 50 *cents a copy:* "The History of Chemistry," Professor John Johnston; "The Centennials of Herman Von Helmholtz and Rudolf Virchow: 'Hermann Von Helmholtz,' Professor Louis Karpinski; 'Rudolf Virchow—Pathologist,' Dr. Carl Vernon Walker; 'Rudolf Virchow—Anthropologist and Archeologist,'" Professor Arthur E. R. Boak; "The Biology of Death—the Inheritance of Duration of Life in Man," Professor Raymond Pearl; "Vitamins and Food Deficiency Diseases," Dr. Alfred C. Reed; "Fishing in Lake Michigan," Professor A. S. Pearse.

Torreya, for May-June; *New York Botanical Garden, Bronx Park, New York* $1.00 *per year*, 30 *cents a copy:* "The Phyllotaxy of Phoenix," Cornelius B. Bradley; "Teaching the Evolution of Land Plants," B. W. Wells.

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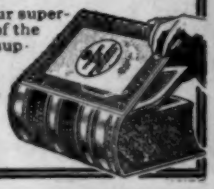

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BOOKS RECEIVED.

Nature Study-Agriculture, William T. Skilling, Normal School, San Diego, Calif. Pages VII plus 332, 13½×18½ cm. Cloth. $1.68, 1920. World Book Co., Yonkers-On-Hudson, N. Y.

Philosophy And The New Physics, Louis Rogier, Translated by Morton Masius, Worcester Polytechnic Institute, Pages XV plus 159, 12½×18½ cm. Cloth. 1921. $1.75. P. Blakiston's Son & Co., Philadelphia.

Chemical Reactions and Their Equations, Ingo W. D. Hackh, San Francisco. Pages VIII plus 138, 12½×18½ cm. Cloth, 1921. $1.75. P. Blakiston's Son & Co., Philadelphia.

Making A High School Program, Myron W. Richardson, Head Master Girls' High School, Boston, Mass. Pages VII plus 27. 12×18 cm. Paper. 1921. 75 cents. World Book Co., Yonkers-On-Hudson, N. Y.

The Quaternary Geology of the New Haven Region Connecticut, Freeman Ward, University of South Dakota, 78 pages. 15×22½ cm. Paper. 1920. Published by the State, Hartford, Conn.

Drainage Modifications and Glaciation In the Danbury Region, Connecticut, Ruth S. Harley, 14½×22 cm. Paper. 1920. Published by the State, Hartford, Conn.

Manual for General Science in the Ninth Grade, Cleveland Public Schools, Ellis Persing, Chairman. 15 Pages. 17×24 cm. Paper. 1920. Board of Education, Cleveland, Ohio.

The Derivation and Standardization of a Series of Diagnostic Tests for the Fundamentals of First Year Algebra, Harl Roy Douglas, University of Oregon, 48 pages, 17×22½ cm. Paper. 1921. Published by the University at Eugene, Ore.

Qualitative Chemical Analysis of Inorganic Substances, by Olin F. Lower, Adelbert College. Pages XV+89. 16×23.5 cm. Cloth. 1921. $1.50 net. P. Blakiston's Son & Co., Philadelphia, Pa.

Comrades in Play, by Community Service, Inc. 84 pages. 10½×16½ cm. Paper. 1920. Community Service. No. 1 Madison Ave., New York City.

Rope Work, by Louis M. Roehl, Cornell University. 47 pages. 15½×23 cm. Paper. 1920.

Regent's Questions and Answers in Physics. Regent's Publishing Company, 32 Union Square, New York City. 13½×16½ cm. Paper. 112 pages.

Regent's Questions and Answers in Chemistry, Regent's Publishing Company, 32 Union Square, New York City. 13½×16½ cm. Paper. 112 pages.

Pattern Making, by Edward M. McCracken, Johns School, Scranton, and Charles H. Sampson, Hamilton School, Boston. Pages VIII plus 118, 27½×20½ cm. Cloth. 1921. D. Van Nostrand Company, New York City.

A Textbook of Organic Chemistry. by Joseph S. Chamberlain, Massachusetts Agricultural College. Pages XLIII+959. 14.5×20 cm. Cloth. 1921. $4.00 net. P. Blakiston's Son & Co., Philadelphia, Pa.

School Arithmetics, Three-Book Series, by George Wentworth and David E. Smith, Teachers College, Columbia University. Pages, Book One, VI+282+16; Book Two, VI+298+19; Book Three, VI+346+19. 14×19 cm. each. Cloth. 1920. 72 cents, 76 cents and 92 cents respectively. Ginn & Co., Boston, Mass.

Report of Division of Educational Test for '19-'20, by Walter S. Monroe, University of Illinois. 64 pages. 15.5×23 cm. Paper. 1920. 25 cents. The University of Illinois, Urbana.

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Check-List of the Insects of Connecticut, by Wilton E. Button, Experimental Station, New Haven, Conn. 397 pages. 15×23 cm. Paper. 1920. Published by the State, Hartford.

Junior High School Mathematics, by Walter W. Hart, University of Wisconsin. Pages IX+226. 12×18 cm. Cloth. 1921. D. C. Heath & Co., Chicago.

Business Mathematics, by Edward I. Edgerton, Dickinson High School, Jersey City, and Wallace E. Bartholomew, New York State State Board of Regents. Pages VI+305. 13×195 cm. Cloth. 1921. The Ronold Press Company, New York City.

BOOK REVIEWS.

Making a High School Program, Myron W. Richardson, Girls' High School, Boston, Mass. Pages 7×27. Kraft Binding. 1921. 75 cents. The World Book Co., Yonkers, N. Y.

Without question the most difficult matter in the government of a large high school is that of making the program for each semester's work. It's a job which only a person thoroughly familiar with all of the courses and details can successfully make, and even then the principal needs all the assistance from the faculty that he possibly can secure. There are very few laymen who appreciate the tremendous task of constructing a successful program for a school of more than 1,000 pupils.

This little book offers information which can be used in any high school. It is filled with real concrete aids which are helpful to every high school principal. There are many schemes that have been offered by principals and assistants who have to deal with the making of programs in large high schools, but the schemes presented here appear to the writer to be capable of being put into successful execution in the formation of the program better than any that have ever been brought to his attention.

In consideration of the construction of the program as given by the author it states:

"1. The program must give the individual what he needs and wants.

"2. It must distribute the recitation and study periods throughout the building so as to secure practically constant use, having no room overcrowded and no room idle during the school days.

"3. It must keep the teachers evenly employed.

"4. It must avoid friction of classes, teachers and rooms.

"5. It must be planned to use the entire school plant with the most efficiency and with the least expense."

There are given facsimiles of forms that the author uses in forming a program in the school mentioned above, but perhaps one of the strongest points of the book is its clear exposition of many of the principles involved in the high school program making.

To the young principal it will be especially helpful; to the person going from a smaller to a larger school it will be especially helpful; and to the experienced principal it undoubtedly will bring many new and helpful suggestions. It is a little book that every high school principal ought to possess, whether they believe all of the points or not. C. H. S.

Regent's Questions and Answers in Physics, Regent's Publishing Co., 32 Union Square, New York City. 112 pages 13½×16½ cm. Paper. 60 cents.

What is said with reference to this book is equally applicable to the one in chemistry described below. This is a compilation of questions in physics from the New York State examinations questions. It makes a splendid book in the hands of every physics teacher for tests in his subject.

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The answers to the questions are also given. It is up-to-date in every respect, questions being included from the year 1916. The questions are classified and the authors claim that the answers are correct. A good feature is that the many questions are accompanied by diagrams. It should be in the hands of all secondary physics instructors.

C. H. S.

Comrades In Play, Community Service Inc. 84 pages. 10½×16½ cm. Paper. 1920. Community Service, No. 1, Madison Ave., New York.

This is a little book which is a composition of those ideas which have to deal with the bringing of people, by means of play, into a closer social comradeship. It is especially adapted for those people who wish to make a social gathering interesting by devising games by which company will be amused. It is adaptable for both young and older people. It is a book which ought to be owned by those people who are obliged to engineer social parties.

There are five chapters written in excellent style and easily understood. There is an appendix on the bibliography of play, one on church entertainments, and one on the drama. It is a valuable book in the hands of those people upon whom the success of entertainments depend.

C. H. S.

Rope Work, Louis M. Roehl, Cornell University, 47 pages 23×15½ cm. Paper. 1920. 86 cents. The Bruce Publishing Company, Milwaukee, Wis.

The writer of this little review has been privileged to read several books on rope work, especially in the tying of knots, but he has never seen one written in such a clear manner nor one in which the numerous figures are so clearly presented. The book is especially helpful to the farm boy or sailor, and in fact, to any person who has to deal with the handling of ropes.

There are directions for finishing the ends of rope, for the tying of knots, for the taking of hitches, for the making of splices and other interesting work. It is printed in clear 10 point type on heavy calendered paper, so that all of the 133 half-tones are shown with sharpness. It is a book that all people who have to deal with the handling of ropes should possess.

C. H. S.

The Quaternary Geology of the New Haven Region, Conn., Freeman Ward, University of South Dakota. 78 pages. 15×22½ cm. Paper. 1920. Published by the State, Hartford, Conn.

The area described in this bulletin is located in the south central part of Connecticut bordering upon Long Island Sound. The region applies to the central lowland of the state. This bulletin is the result of the study of only a small area and is based upon observations made in 1909 and 1910. It describes the topography of the area very completely; likewise the nature of the soil

There are several diagrams and many halftones of the topography and construction of the soil and rocks. Very valuable is the bulletin to all geologists.

C. H. S.

American Chemistry, a Record of Achievement, The Basis for Future Progress, by Harrison Hale, Ph. D., Head of Department of Chemistry, University of Arkansas, First Edition. Pages 7+215. 63 illustrations. Cloth. 1921. D. Van Nostrand Co.

In this volume the author attempts to set forth the more outstanding facts of American Achievement in Chemistry. Its aim is to bring to

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both the general reader, and the reader still in school some notion of the importance of Chemistry in the life of the American people. To this end some record of the more important advances in chemistry to which America has contributed is given and in order that the reader may gain the habit of reading chemical articles in current literature, frequent references are made to such articles.

Among the topics treated are Water and Sewage, Foods, Fertilizers, Textiles, Coal, Tar and Dyes, Fuels, the Silicate Industries, Paints and Varnishes, Rubber, Electrochemistry, Acids and Metals. Splendid brief synopses of these subjects are furnished and the book should be found in every chemistry departmental library. High school pupils doing project work will find it invaluable as a guide whenever the particular project upon which they are working is covered by the book. The glass industry, for example, is excellently treated and any pupil who happens to be studying the preparation of glass will find the chapter on the Silicate Industries very helpful. College students, too, will find the brief chapters of great help as a preliminary to more intensive treatment of the same themes in the larger industrial chemistries.

High school chemistry departments which emphasize the "daily life" side of chemistry should have at least several copies at hand for frequent reference.

Much of the content is recent, especially the parts that deal with progress due to the war. The illustrations are excellent and assist one in understanding the text.

Altogether the little book is suited to fill a real need in connection with chemical instruction. Its spirit is one of stimulation, looking toward further research and achievement in American chemistry.

F. B. W.

Drainage Modifications in the Danbury Region, Connecticut, by Ruth S. Harvey. 59 pages. 14×22½ cm. Paper. 1920. Published by the State, Hartford, Conn.

A very interesting bulletin that has been brought out by the geological survey of the State of Connecticut. This state is one of the very few in the union which is making a complete survey of this kind. And this particular survey is of peculiar interest to geologists because of the many geographical and geological features described. It has to do with special reference to the history of its streams and the effects of glaciation. It is a very helpful pamphlet to all instructors in geology.

C. H. S.

Pattern Making, Edward M. McCracken, Johns School, Scranton, and Charles H. Sampson, Hamilton School, Boston. Pages VIII+118. 27½×20½ cm. Cloth. 1921. D. Van Nostrand Co., New York.

Instructors in high schools who have charge of the woodworking department will be pleased that this book has been written because it's put up in such a manner, not only to appeal to them but it will be a valuable addition to pupils who may be in their classes. The book is written in such a clear and distinct way that pupils in pattern making will have no difficulty in interpreting the directions and drawings.

The core of the book has been to bring out a definite principle in each one of the patterns here described. The patterns are not as easy as they might be, nor as difficult, but a happy medium has been struck so that the pupil will have a little difficult in carrying out the idea, yet, it is not so difficult as to discourage him. Descriptions of tools to be used are given. The book is especially valuable for technical and vocational schools.

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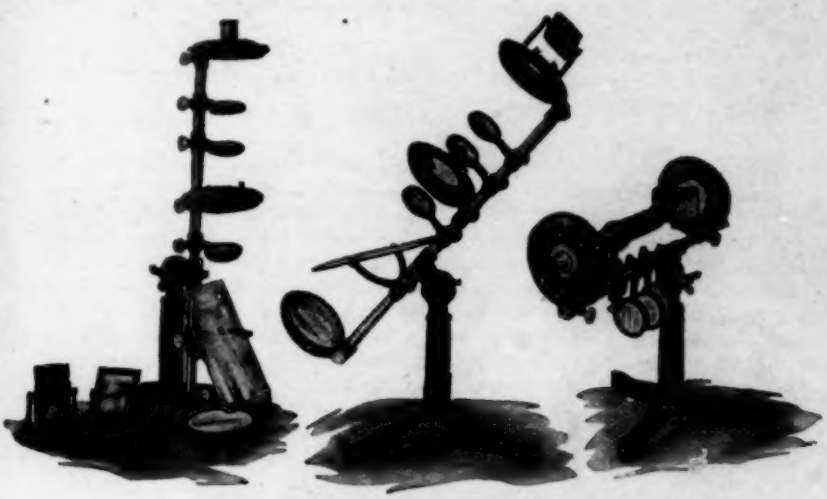

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There are 85 figures and 30 plates. Major paragraphs in bold faced type give the key to the matter discussed in those paragraphs. The book should be in every pattern making shop.

C. H. S.

Manual for General Science in the Ninth Grade, Cleveland Public Schools Ellis Persing, Chairman. 15 pages. 17×24 cm. Paper. 1920. Board of Education, Cleveland, Ohio.

The general science teachers of the city of Cleveland have surely prepared an outline for the study of general science that is second to none that has ever been issued. They have divided the time to be devoted to the various topics with due consideration to all of them. It is of great help and value to pupils of a large city.

The outline is by no means a finished product but is rather a report of the findings of a strong committee that has been at work for some months to produce as near a perfect course in general science as is possible to make. The authors do not claim to have made a perfect outline, nor covered the entire ground, but have borne down heavily on what they consider the fundamentals.

The little book will surely save the general science teacher time and labor if she will possess herself of a copy and put the suggestions into practice in her classes.

C. H. S.

Philosophy and the New Physics, Louis Rogier, Translated by Morton Masius, Worcester Polytechnic Institute. Pages XI+159, 12½×18½ cm. Cloth. 1921. $1.75. P. Blakiston's Son & Co., Philadelphia.

A splendid little book which every physicist in high school and college should own and study, as it discusses the more recent development of physical theories especially those dealing with relativity and quanta of energy. The author has condensed into this book much valuable information which would require one a long time to accumulate. By a study of the book you may get this information in a comparatively short time. It is written in such language that the ordinary high school teacher of physics would be able to understand. It is worthy of a large sale.

C. H. S.

Nature Study-Agriculture, William T. Skilling, Normal School, San Diego, Calif. Pages VII+332, 13½×18½ cm. Cloth. $1.68. 1920. World Book Co., Yonkers-On-Hudson, N. Y.

This book contains many new and up-to-date methods of presentation in secondary school agriculture. It is written in a style pleasing to the eighth and ninth graders. It contains nothing but what these people will readily understand and at the same time presents all of the essentials of an agricultural course for pupils of the grades mentioned.

Throughout the book there are marginal notes containing the key to the topics discussed opposite these notes. At the end of each chapter, of which there are 18, there is a list of experiments on the work covered in that chapter as well as references to these same topics. Scattered throughout the text are directions for raising various kinds of crops and directions given in such a way that the pupil will be encouraged to carry out the project.

There are 266 drawings and half-tones carefully selected illustrating the points that are being discussed. It is a book which eighth and ninth grade teachers should introduce into their classes.

C. H. S.